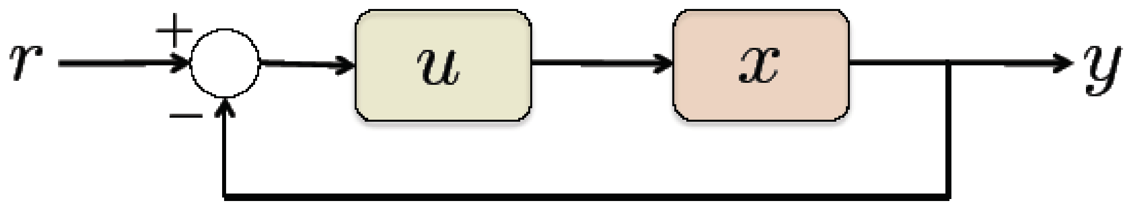
PID Controller

-M. A. El-dosuky

Robotics

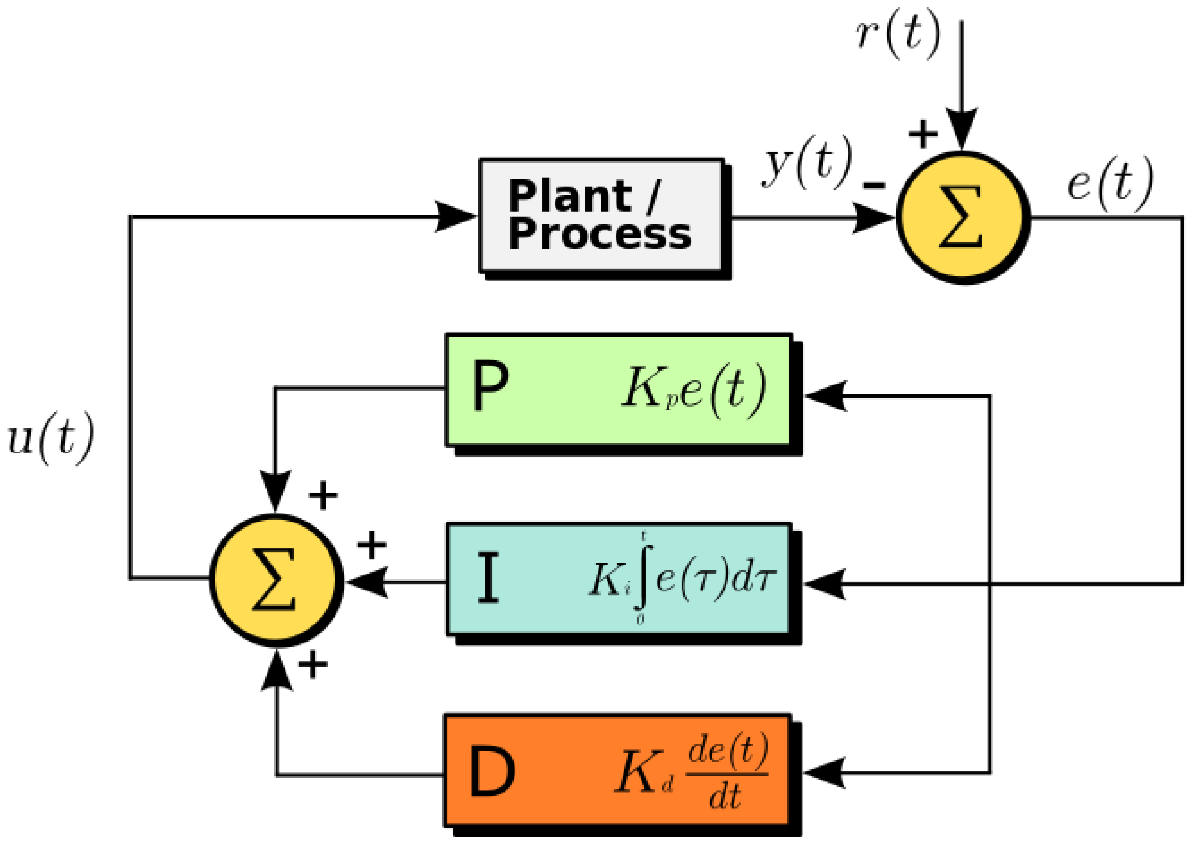
Control blocks

- **State:** is a representation of what the system is doing at a certain time, denoted as *x.*
- **Dynamics:** is a description of how the state changes over time.
- **Reference:** is what we want the system to do, denoted as *r*.
- **Output**: is a measurement of some aspects of the system, denoted as *y.*
- **Input**: is a control signal, denoted as *u*.
- **Feedback**: is mapping from outputs to inputs.



Robotics

PID



*A **plant** is the combination of process and actuator.*

Robotics

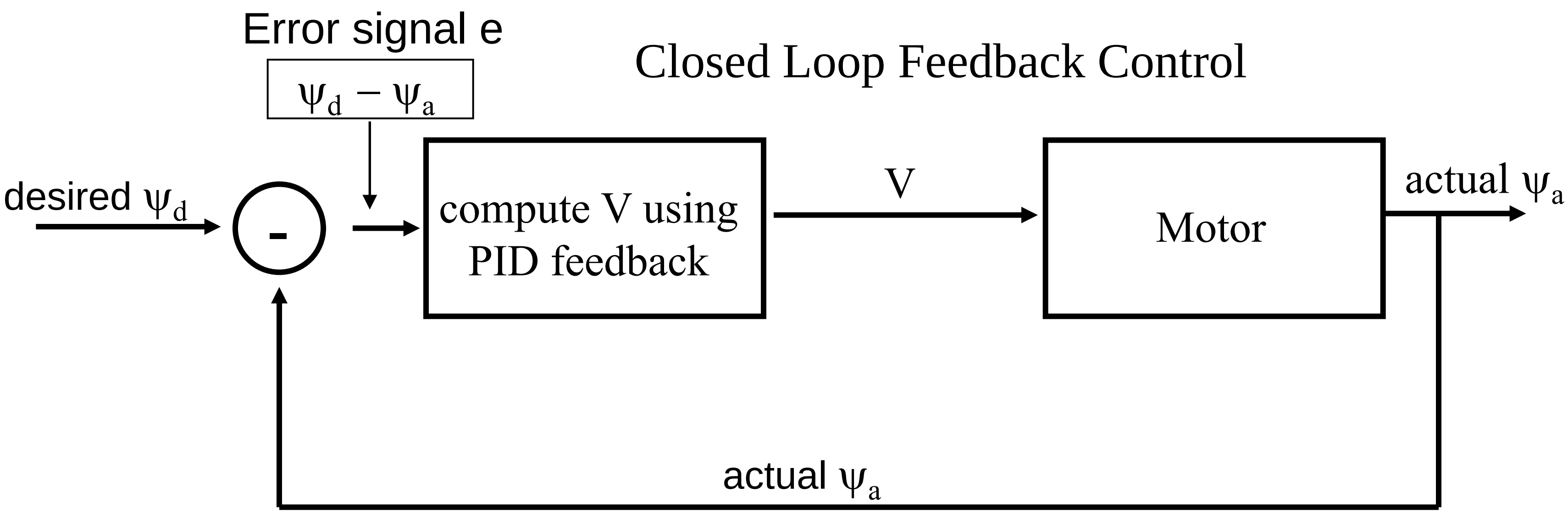
Control Theory

PID controller: Proportional / Integral / Derivative control

$$e = \psi_d - \psi_a$$

$$V = K_p \cdot e + K_i \int e \, dt + K_d \frac{de}{dt}$$

Closed Loop Feedback Control



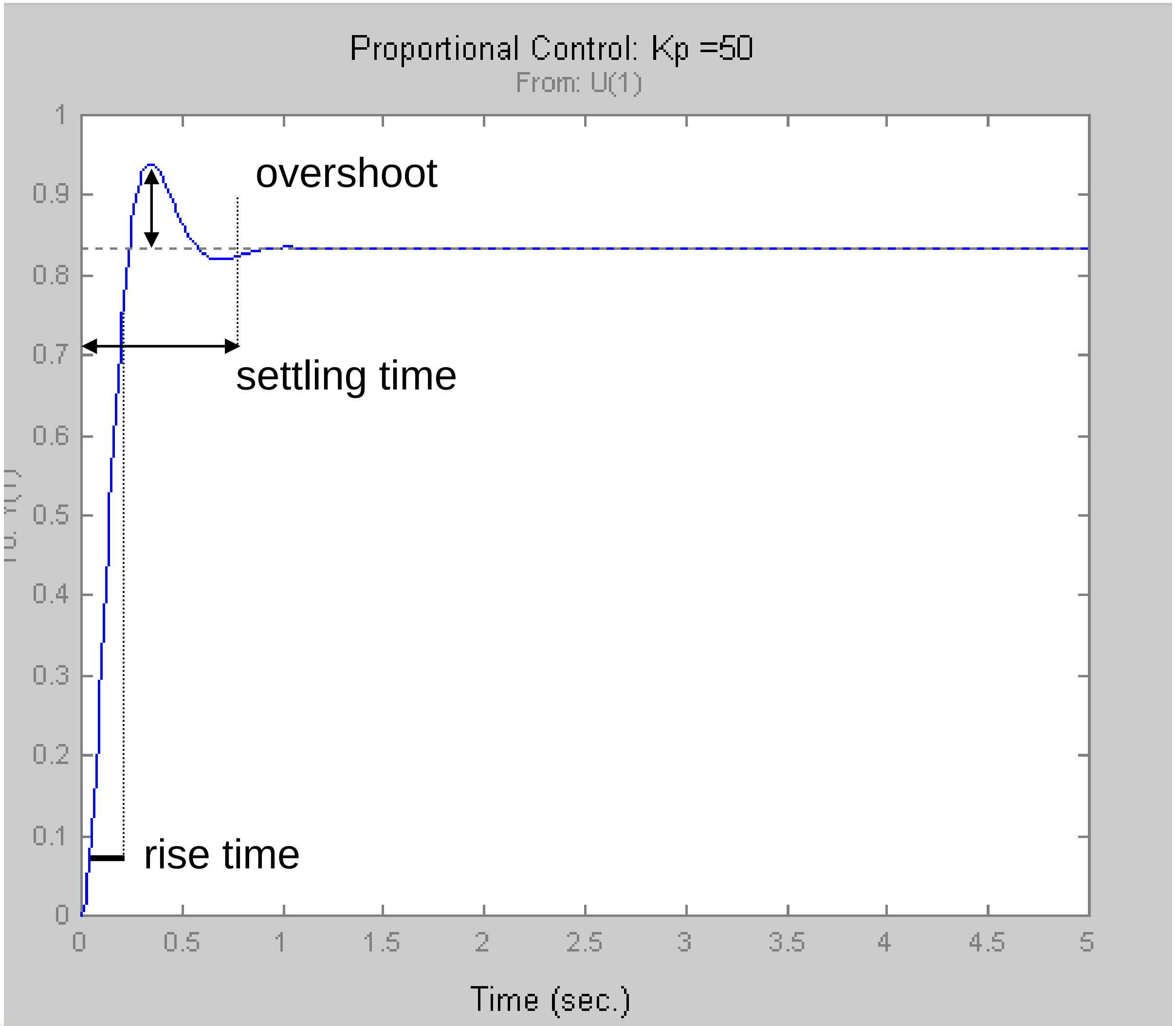
Reference book: Modern Control Engineering, Katsuhiko Ogata, ISBN0-13-060907-2

Robotics

criteria of control

Stability:.
Tracking:.
Robustness:.
Disturbance rejection:.
Optimality:.

Evaluating the response



steady-state error

ss error -- difference from the system's desired value

overshoot -- % of final value exceeded at first oscillation

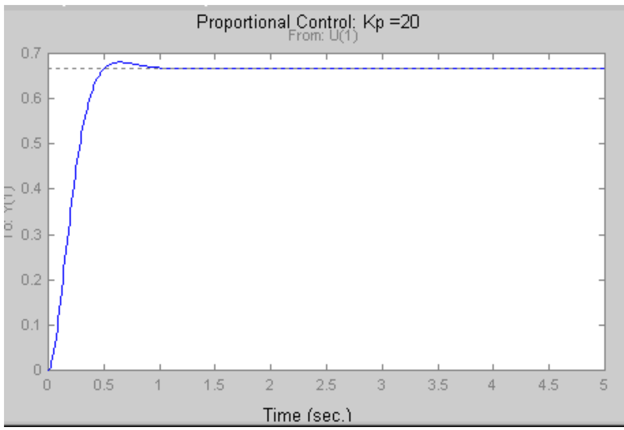
rise time -- time to span from 10% to 90% of the final value

settling time -- time to reach within 2% of the final value

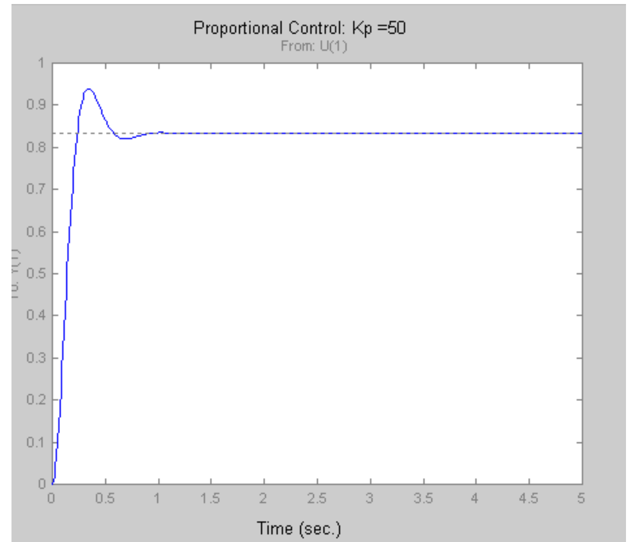
How can we eliminate

the steady-state error?

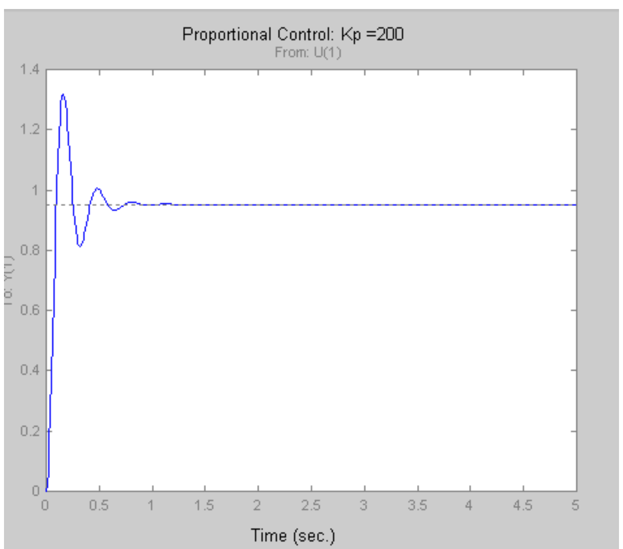
Control Performance, P-type



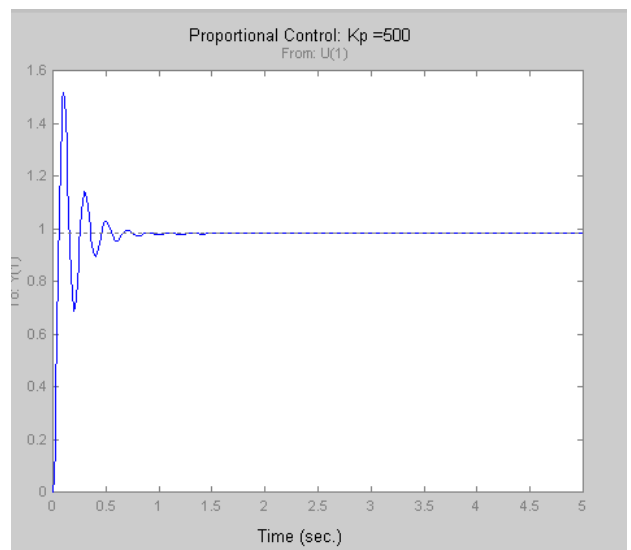
$K_p = 20$



$K_p = 50$



$K_p = 200$

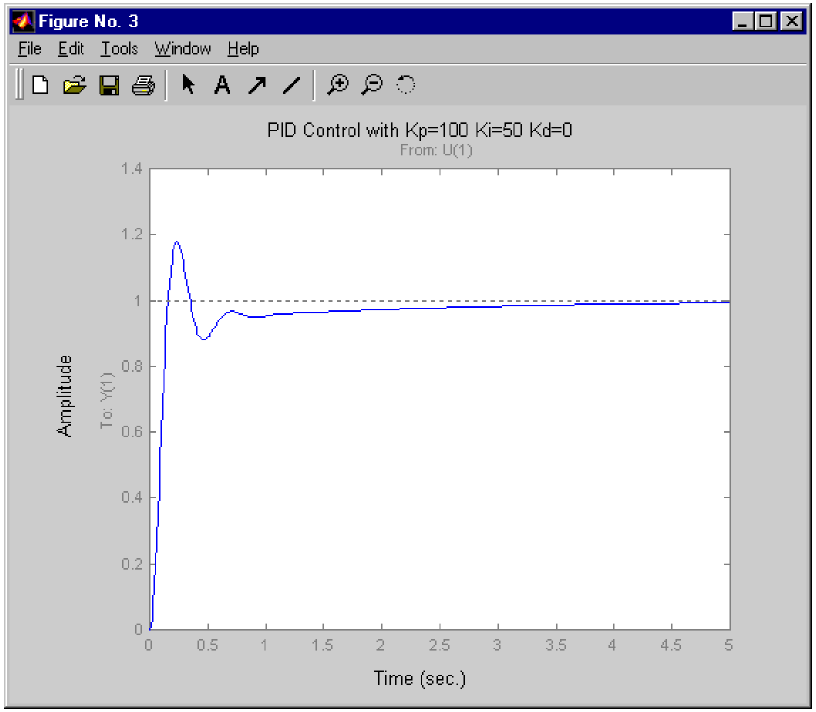


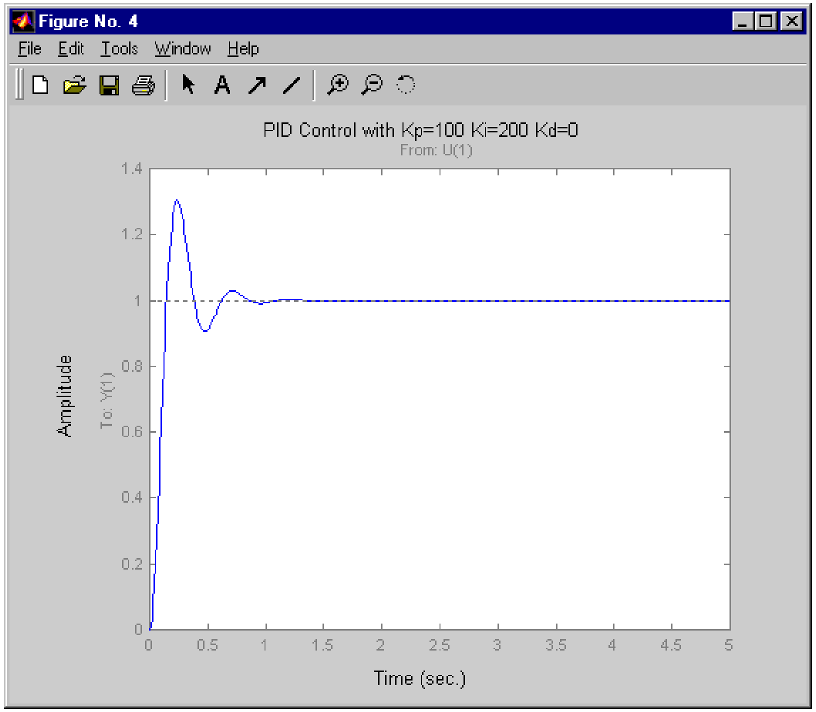
$K_p = 500$

Robotics

Control Performance, PI - type

$K_p = 100$



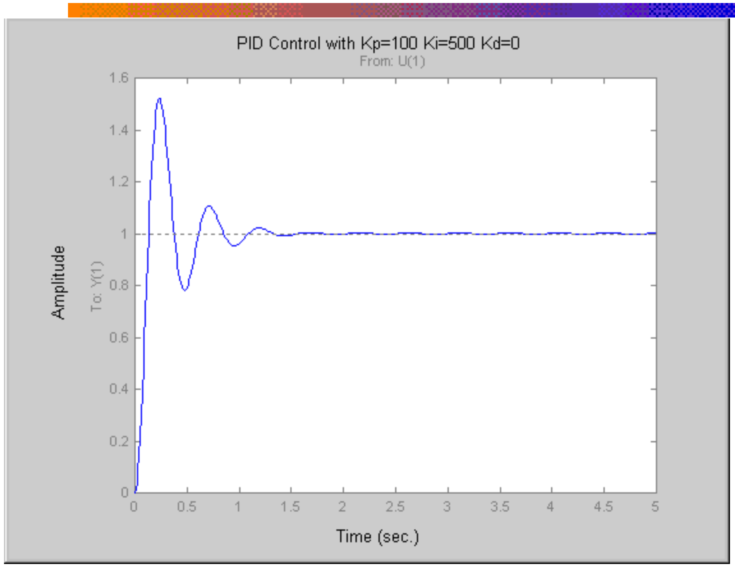


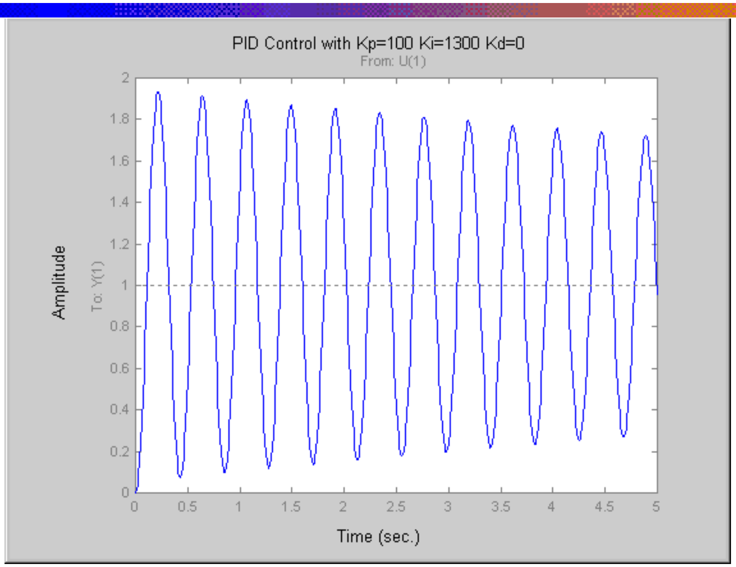
$K_i = 50$

$K_i = 200$

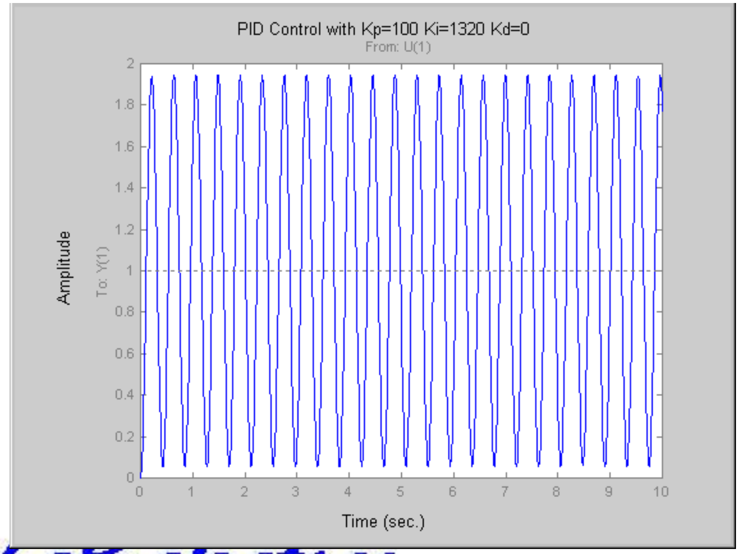
Robotics

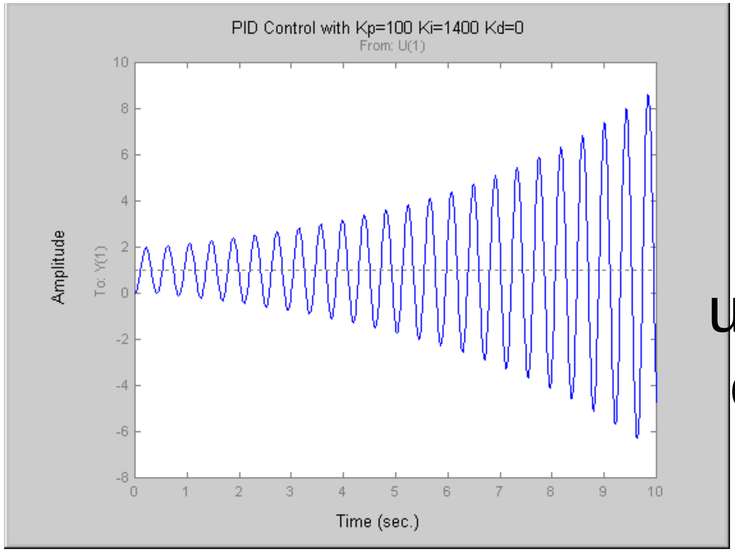
You’ve been integrated...





$K_p = 100$





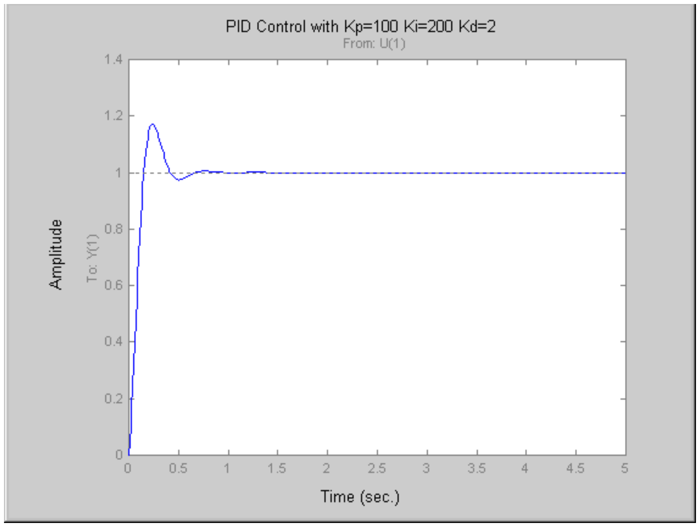
unstable & oscillation

Control Performance, PID-type

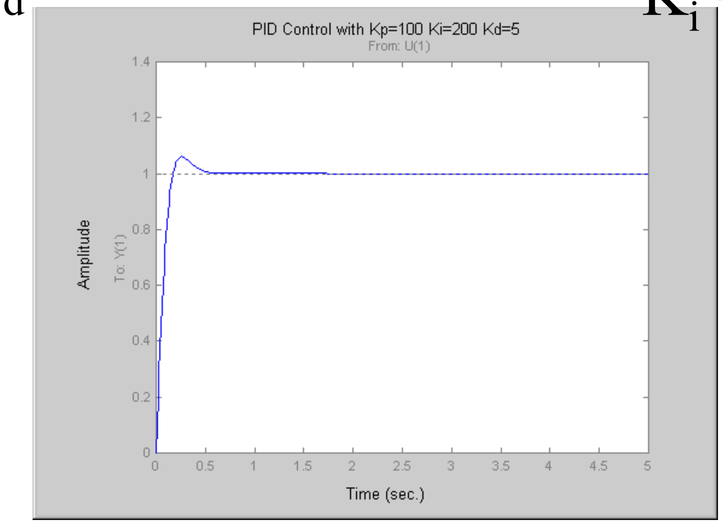
$K_p = 100$

$K_i = 200$

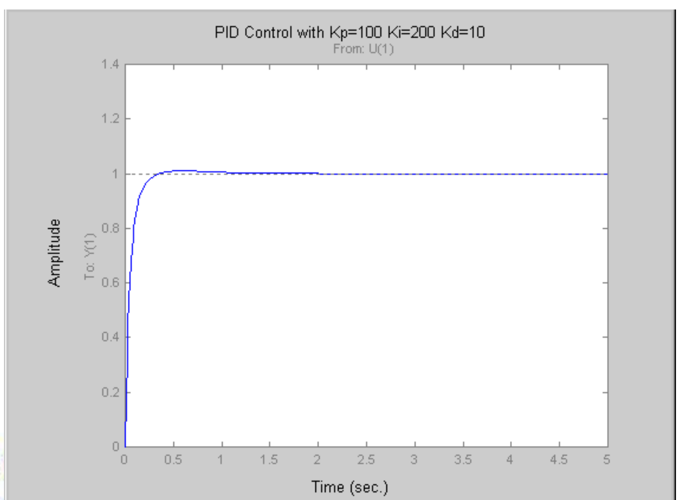
$K_d = 2$



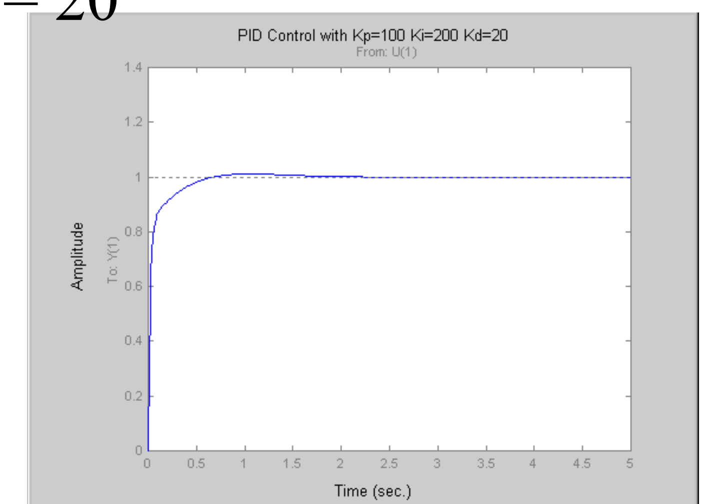
$K_d = 5$



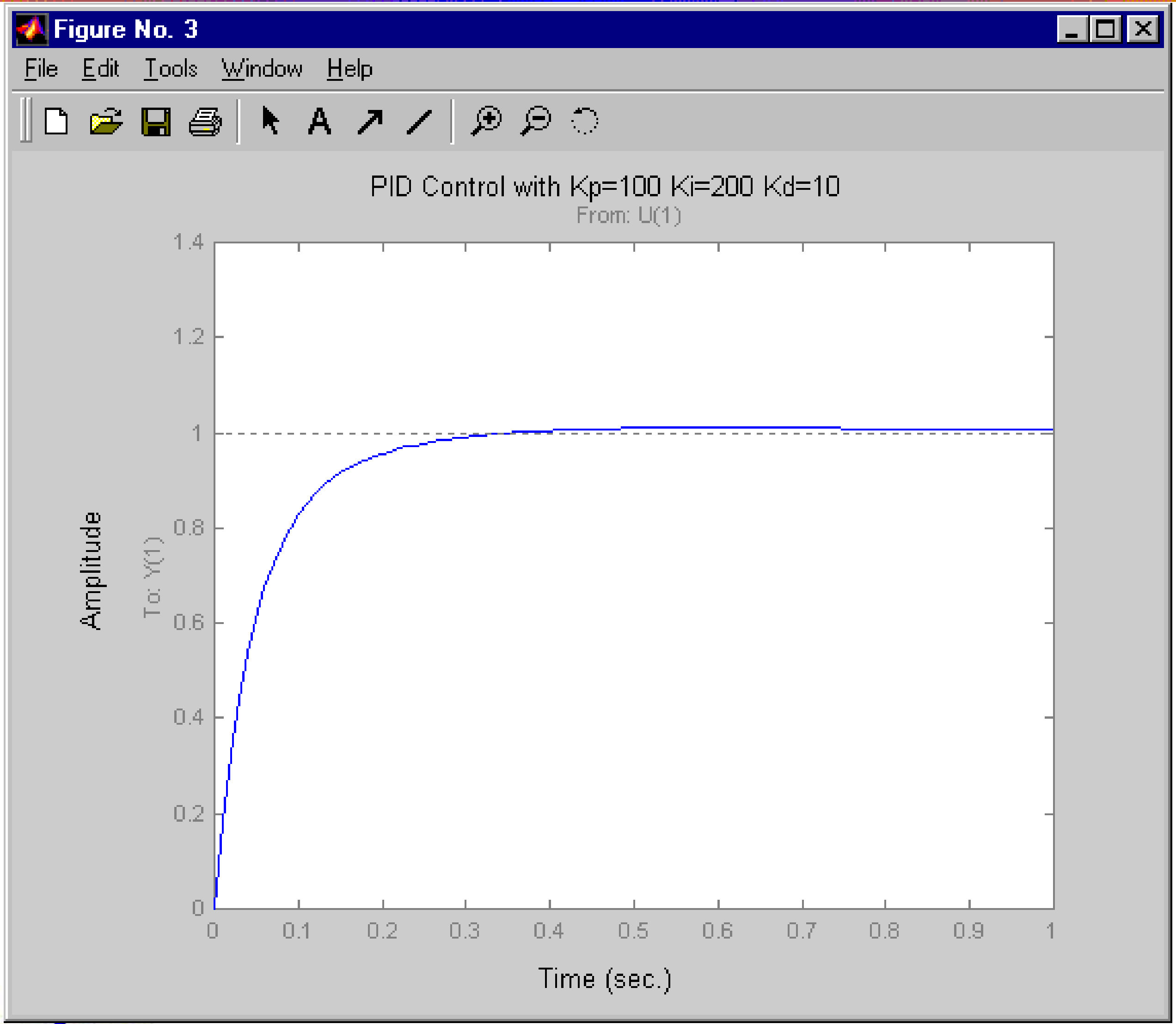
$K_d = 10$



$K_d = 20$



PID final control



Robotics

Linear Control Theory

- Linear Control System
  - State space equation of a system

$$\dot{x} = Ax + Bu \qquad \text{(Equ. 1)}$$

  - Example: a system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

  - Eigenvalue of A are the root of characteristic equation

$$|\lambda I - A| = 0 \qquad |\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 0 & \lambda \end{vmatrix} = \lambda^2 = 0$$

  - Asymptotically stable $\Leftrightarrow$ all eigenvalues of A have negative real part

Robotics

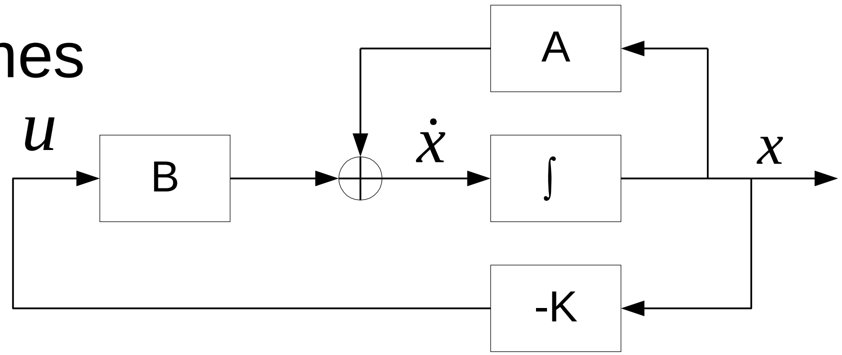
Linear Control Theory

– Find a state feedback control $u = -K \cdot x$ such that the closed loop system is asymptotically stable

$$u = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \text{(Equ. 2)}$$

– Closed loop system becomes

$$\dot{x} = (A - BK)x$$



– Chose K, such that all eigenvalues of A'=(A-BK) have negative real parts

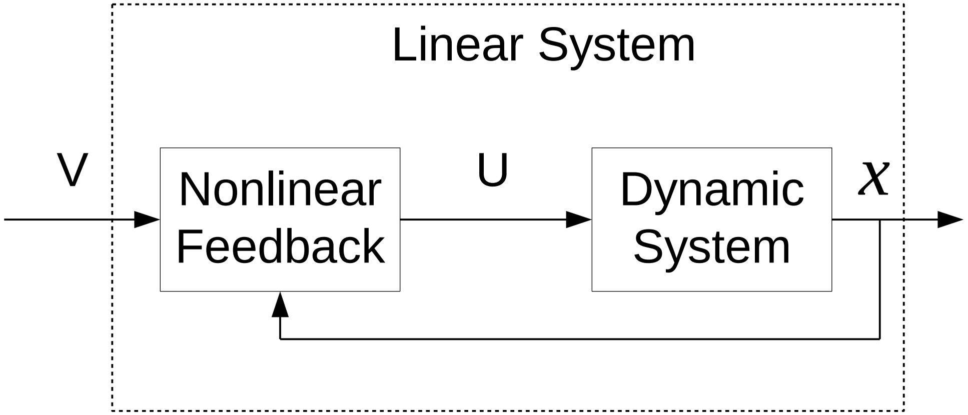
$$|\lambda I - A'| = \begin{vmatrix} \lambda & -1 \\ k_1 & \lambda + k_2 \end{vmatrix} = \lambda^2 + k_2\lambda + k_1 = 0$$

Linear Control Theory

- Feedback linearization
  - Nonlinear system $\dot{X} = f(x) + G(x)U$

$$U = [-G^{-1}(x)f(x) + G^{-1}(x)V]$$

$$\dot{X} = V$$



  - Example:

Original system:

$$\ddot{x} + \cos x = U$$

Nonlinear feedback:

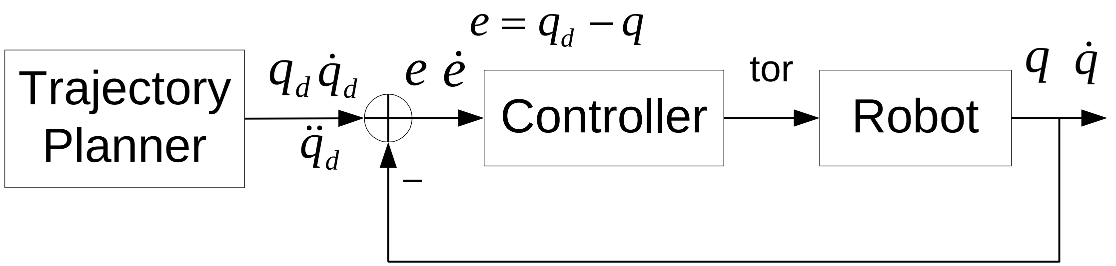
$$U = \cos x + V$$

Linear system: $\ddot{x} = V$

Robotics

Robot Motion Control (I)

- Joint level PID control
  - each joint is a servo-mechanism
  - adopted widely in industrial robot
  - neglect dynamic behavior of whole arm
  - degraded control performance especially in high speed
  - performance depends on configuration



Robot Motion Control (II)

- Computed torque method
  - Robot system: $\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$
  - Controller:

$$tor = D(q)[\ddot{q}^d + k_v(\dot{q}^d - \dot{q}) + k_p(q^d - q)] + H(q,\dot{q}) + C(q)$$

$$(\ddot{q}^d - \ddot{q}) + k_v(\dot{q}^d - \dot{q}) + k_p(q^d - q) = 0$$

How to chose Kp, Kv ?

Error dynamics $$\ddot{e} + k_v\dot{e} + k_p e = 0$$

Advantage: compensated for the dynamic effects

Condition: robot dynamic model is known

Robotics

Robot Motion Control (II)

How to chose Kp, Kv to make the system stable?

Error dynamics $\ddot{e} + k_v \dot{e} + k_p e = 0$

Define states: $x_1 = e$, $x_2 = \dot{e}$ $\Rightarrow$ $\dot{x}_1 = x_2$, $\dot{x}_2 = -k_v x_2 - k_p x_1$

In matrix form: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_p & -k_v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = AX$

Characteristic equation: $|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ k_p & \lambda + k_v \end{vmatrix} = \lambda^2 + k_v \lambda + k_p = 0$

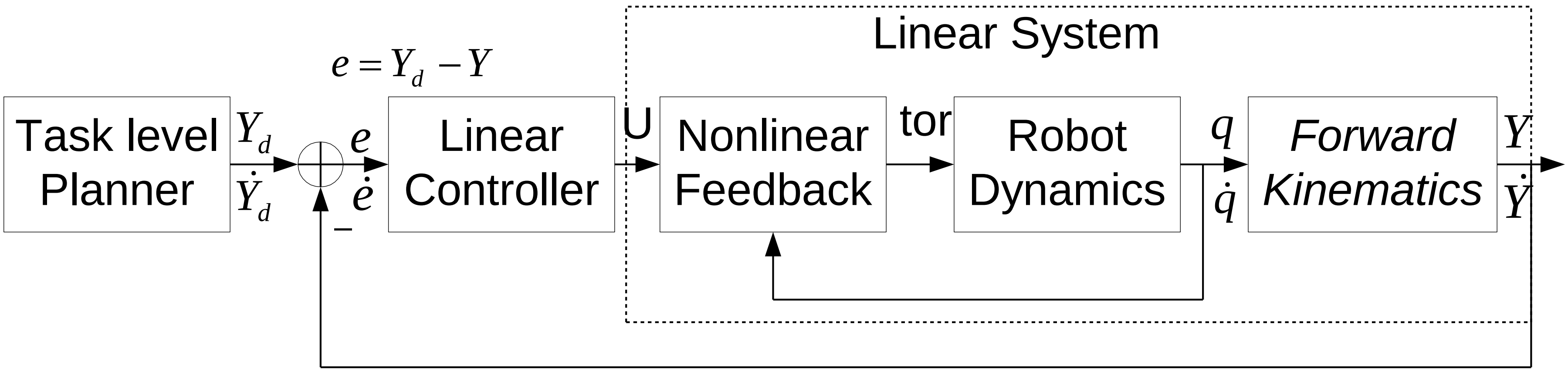
The eigenvalue of A matrix is: $\lambda_{1,2} = \dfrac{-k_v \pm \sqrt{k_v{}^2 - 4k_p}}{2}$

Condition: λ have negative real part $\Rightarrow$ One of a selections: $k_v > 0$, $k_p > 0$

Robotics

Robot Motion Control (III)

- Non-linear Feedback Control



Robot System:

$$\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$$

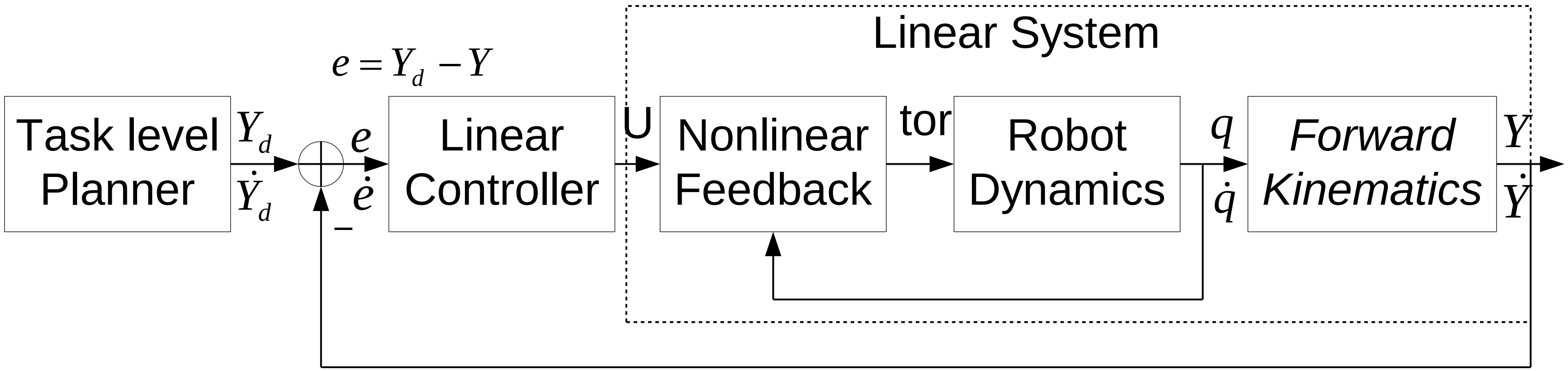
Jocobian: $\dot{Y} = \dfrac{d}{dq}[h(q)] \cdot \dot{q} = J\dot{q} \Longrightarrow \ddot{Y} = \dot{J}\dot{q} + J\ddot{q} \Longrightarrow \ddot{q} = J^{-1}(\ddot{Y} - \dot{J}\dot{q})$

$$D(q)J^{-1}(\ddot{Y} - \dot{J}\dot{q}) + H(q,\dot{q}) + C(q) = \tau$$

Robotics

Robot Motion Control (III)

- Non-linear Feedback Control



Design the nonlinear feedback controller as:

$$tor = D(q)J^{-1}(U - \dot{J}\dot{q}) + H(q,\dot{q}) + C(q)$$

Then the linearized dynamic model:

$$D(q)J^{-1}\ddot{Y} = D(q)J^{-1}U \quad \Rightarrow \quad \ddot{Y} = U$$

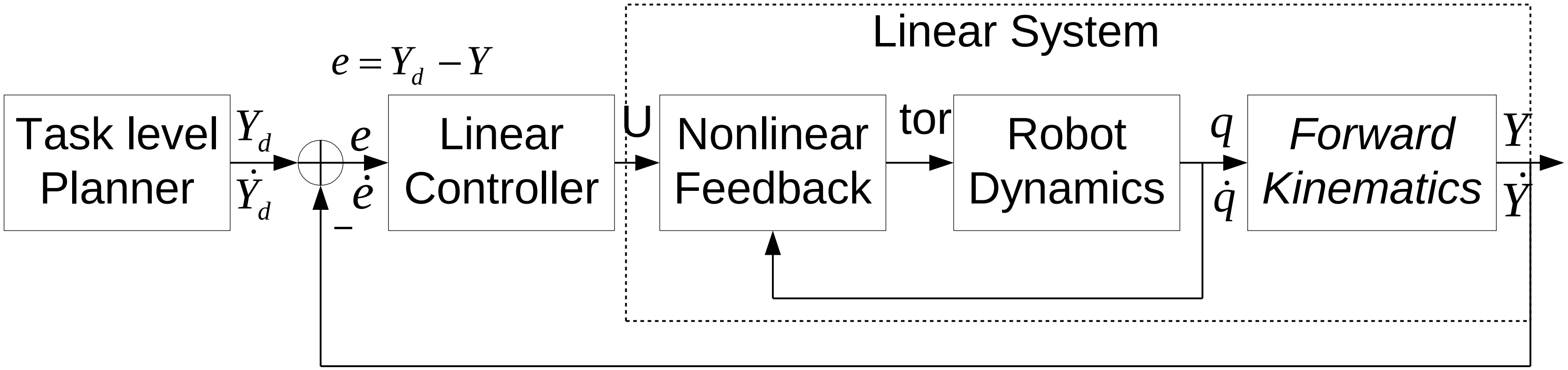
Design the linear controller: $U = \ddot{Y}_d + k_v(\dot{Y}_d - \dot{Y}) + k_p(Y_d - Y)$

Error dynamic equation: $\ddot{e} + k_v\dot{e} + k_p e = 0$

Robotics

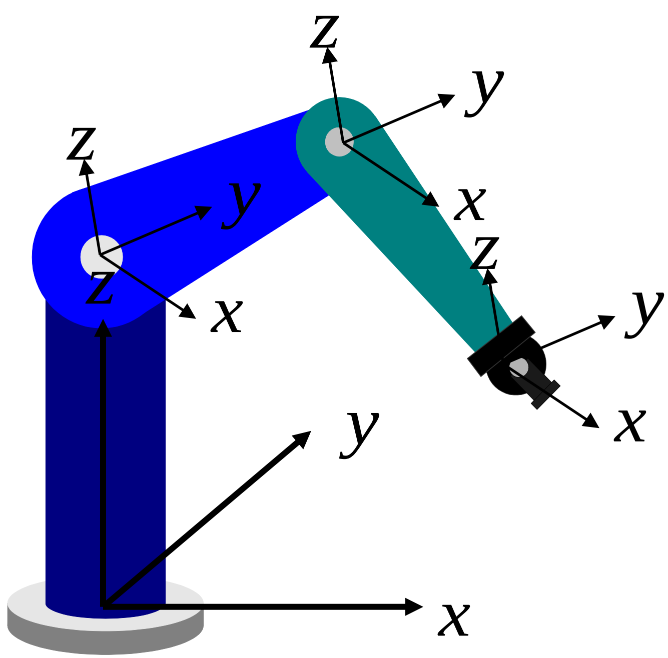
Project

- Simulation study of Non-linear Feedback Control



Robotics

Thank you!



Robotics