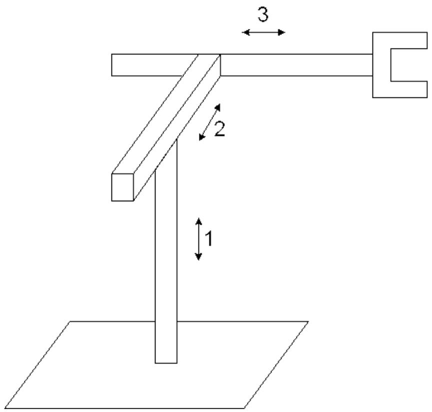
Kinematics II

-M. A. El-dosuky

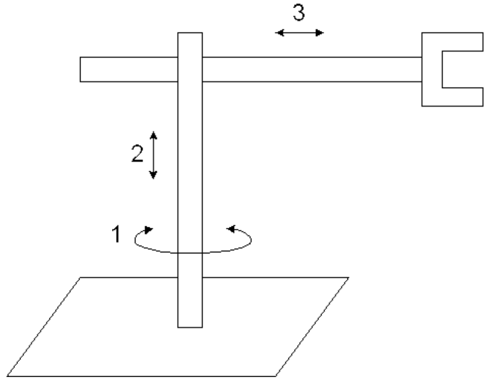
Robotics

Review

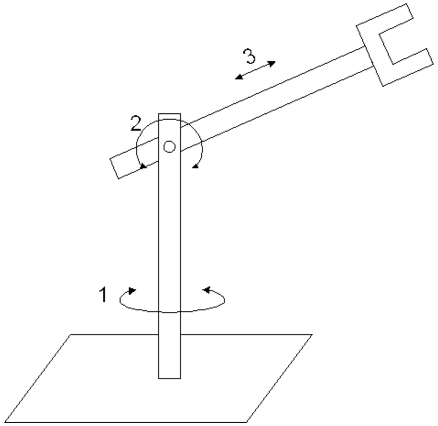
- Manipulators:



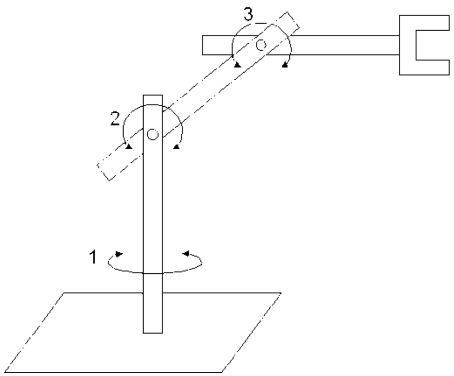
Cartesian: PPP



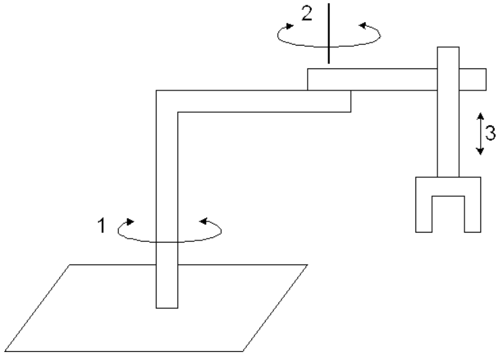
Cylindrical: RPP



Spherical: RRP

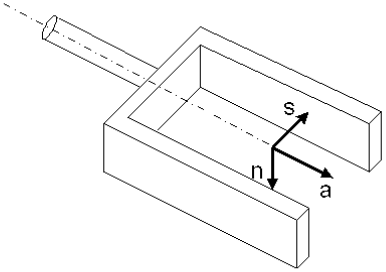


Articulated: RRR



SCARA: RRP

(Selective Compliance Assembly Robot Arm)



Hand coordinate:

n: normal vector; **s**: sliding vector;

a: approach vector, normal to the tool mounting plate

Basic Rotation Matrices

– Rotation about x-axis with θ

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$

– Rotation about y-axis with θ

$$Rot(y,\theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

– Rotation about z-axis with θ

$$Rot(z,\theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{xyz} = RP_{uvw}$$

Robotics

Review

- Composite Homogeneous Transformation Matrix
- Rules:
  - Transformation (rotation/translation) w.r.t. (X,Y,Z) (OLD FRAME), using **pre-multiplication**
  - Transformation (rotation/translation) w.r.t. (U,V,W) (NEW FRAME), using **post-multiplication**

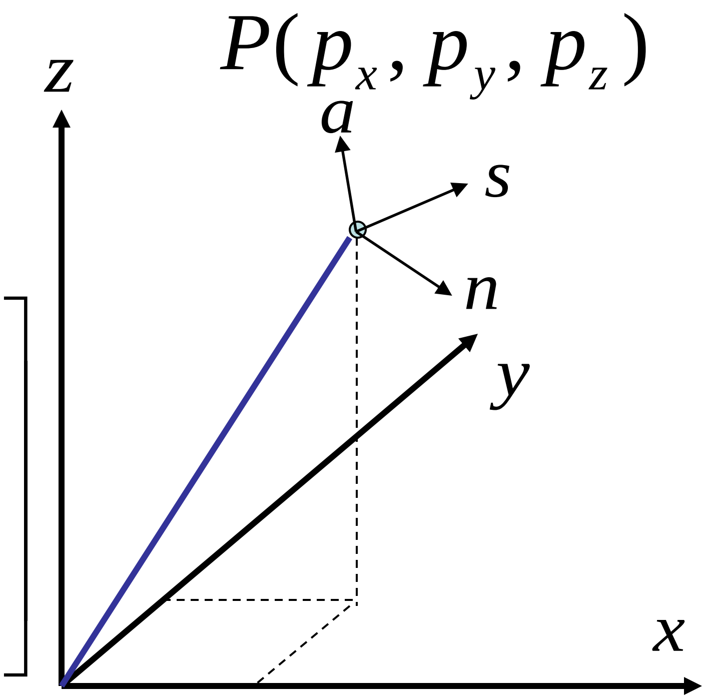
Review

- Homogeneous Representation
  - A point in R^3 space

$$P = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$ ← Homogeneous coordinate of P w.r.t. OXYZ

  - A frame in R^3 space

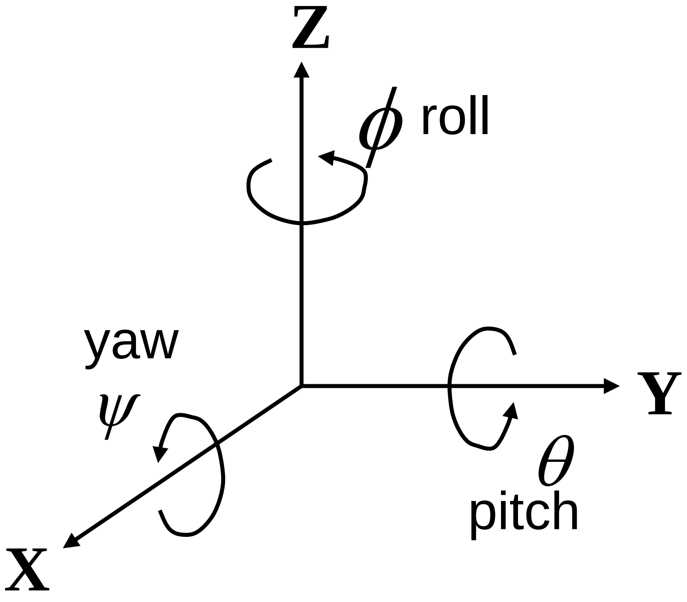
$$F = \begin{bmatrix} n & s & a & P \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Robotics

Review

- Orientation Representation (Euler Angles)
  - Description of Yaw, Pitch, Roll
    - A rotation of ψ about the OX axis ($R_{x,\psi}$) -- yaw
    - A rotation of θ about the OY axis ($R_{y,\theta}$) -- pitch
    - A rotation of ϕ about the OZ axis ($R_{z,\phi}$) -- roll



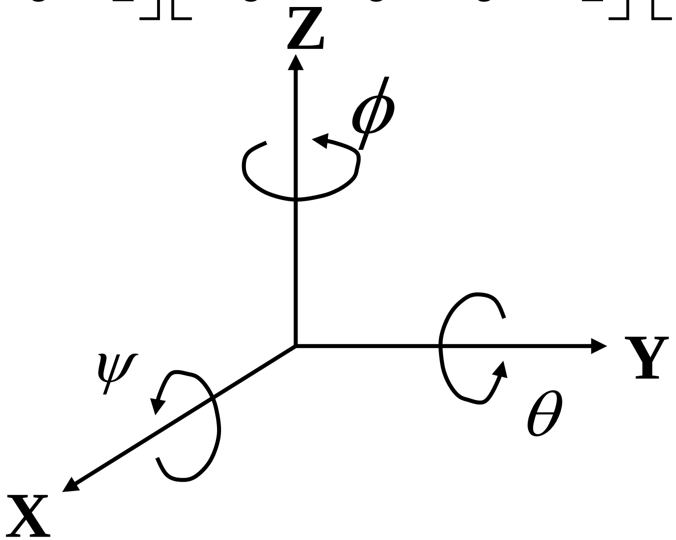
Robotics

Quiz 1

- How to get the resultant rotation matrix for YPR?

$$T = R_{z,\phi} R_{y,\theta} R_{x,\psi}$$

$$= \begin{bmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

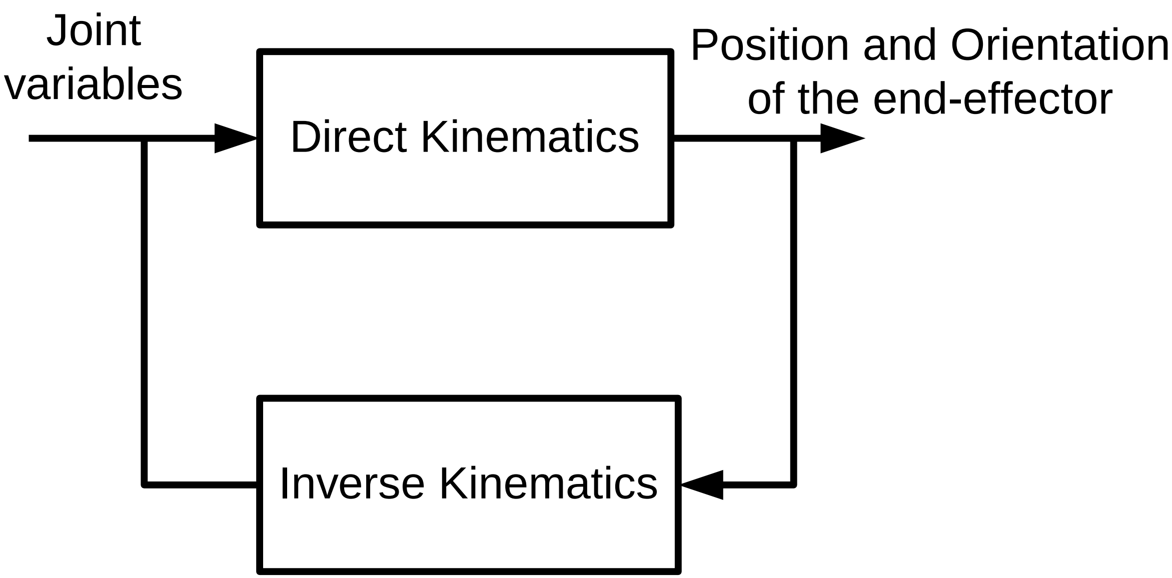


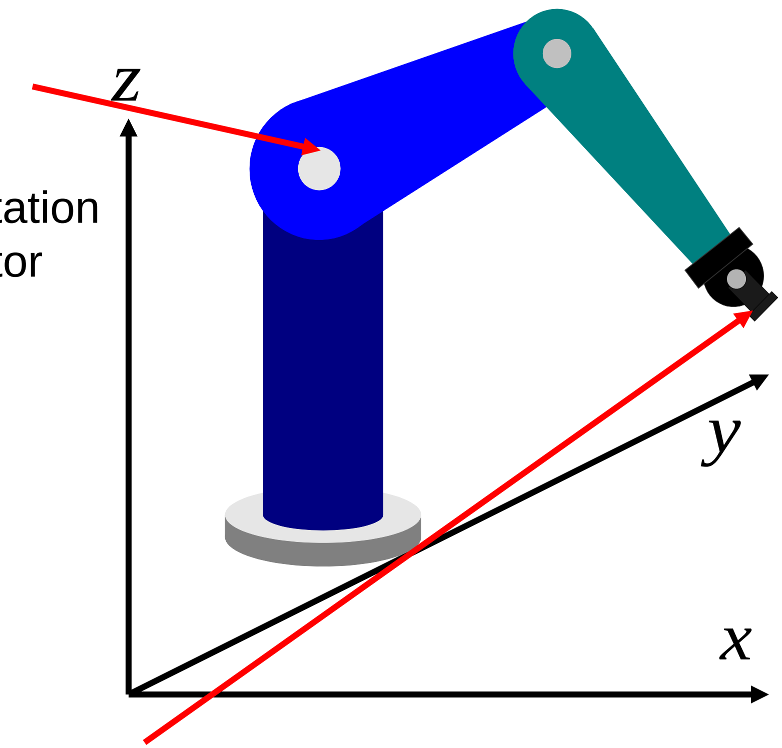
Robotics

Kinematics Model

- Forward (direct) Kinematics

$$q = (q_1, q_2, \cdots q_n)$$





$$Y = (x, y, z, \phi, \theta, \psi)$$

- Inverse Kinematics

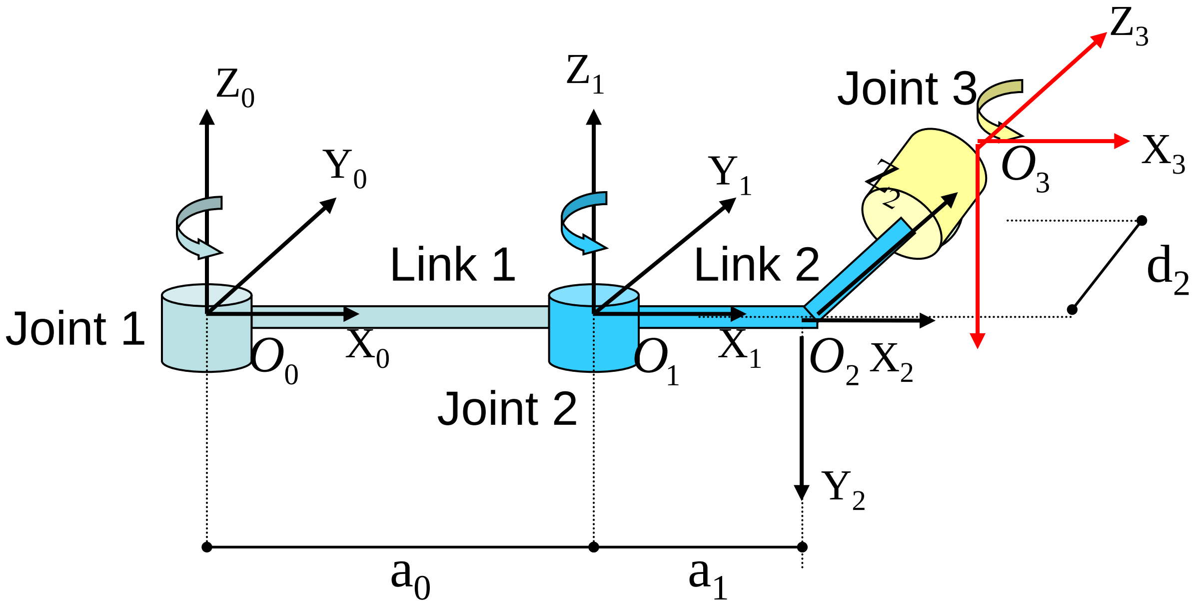
Robotics

Denavit-Hartenberg Convention

- Number the joints from 1 to n starting with the base and ending with the end-effector.
- *Establish the base coordinate system.* Establish a right-handed orthonormal coordinate system (X_0, Y_0, Z_0) at the supporting base with Z_0 axis lying along the axis of motion of joint 1.
- *Establish joint axis.* Align the Z_i with the axis of motion (rotary or sliding) of joint i+1.
- *Establish the origin of the ith coordinate system.* Locate the origin of the ith coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- *Establish X_i axis.* Establish $X_i = \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$ or along the common normal between the Z_{i-1} & Z_i axes when they are parallel.
- *Establish Y_i axis.* Assign $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$ to complete the right-handed coordinate system.
- Find the link and joint parameters

Example I

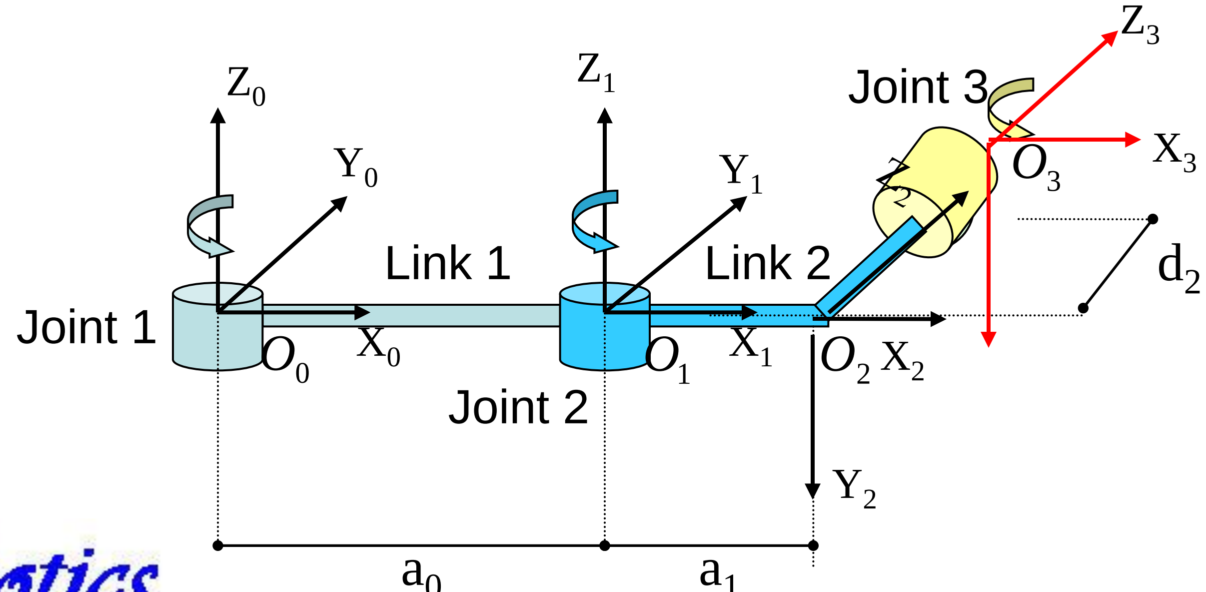
- 3 Revolute Joints



Robotics

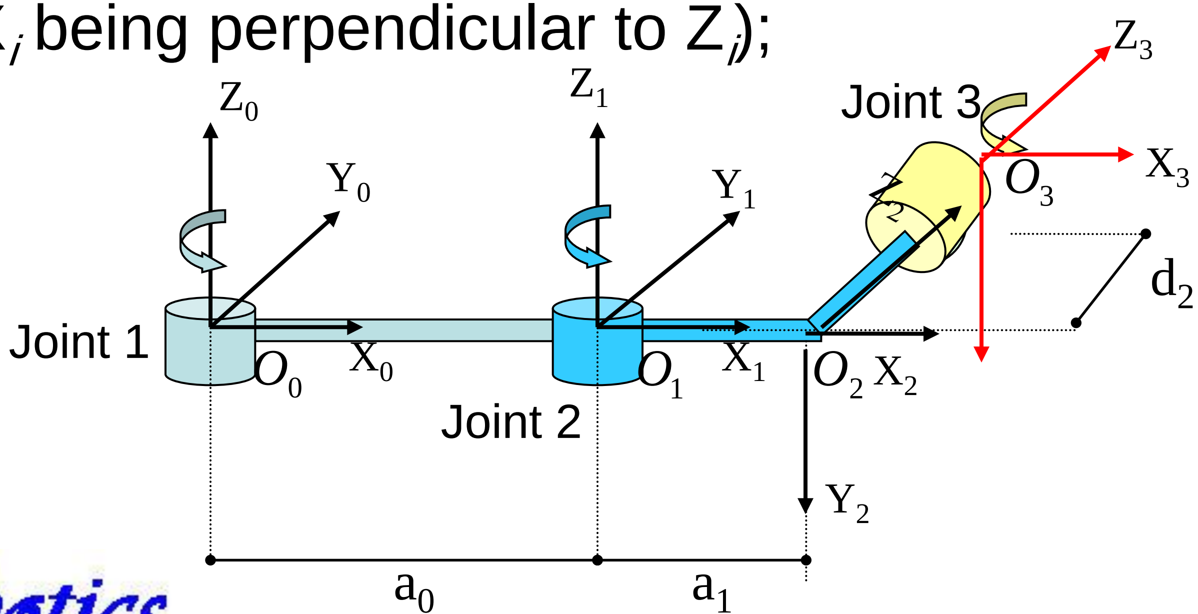
Link Coordinate Frames

- *Assign Link Coordinate Frames:*
  - To describe the geometry of robot motion, we assign a Cartesian coordinate frame (O_i, X_i,Y_i,Z_i) to each link, as follows:
    - establish a right-handed orthonormal coordinate frame O_0 at the supporting base with Z_0 lying along joint 1 motion axis.
    - the Z_i axis is directed along the axis of motion of joint (i + 1), that is, link (i + 1) rotates about or translates along Z_i;



Link Coordinate Frames

- Locate the origin of the ith coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- the X_i axis lies along the common normal from the Z_{i-1} axis to the Z_i axis $X_i = \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$, (if Z_{i-1} is parallel to Z_i, then X_i is specified arbitrarily, subject only to X_i being perpendicular to Z_i);

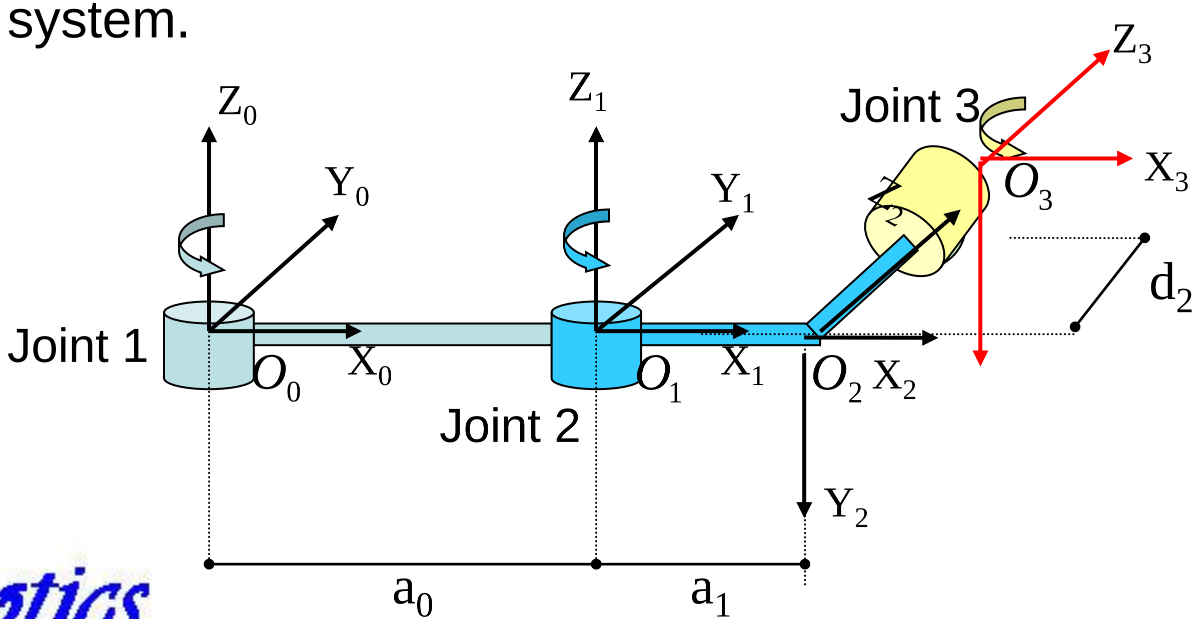


Robotics

Link Coordinate Frames

– Assign $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$ to complete the right-handed coordinate system.

- The hand coordinate frame is specified by the geometry O_n of the end-effector. Normally, establish Z_n along the direction of Z_{n-1} axis and pointing away from the robot; establish X_n such that it is normal to both Z_{n-1} and Z_n axes. Assign Y_n to complete the right-handed coordinate system.



Link and Joint Parameters

- *Joint angle* θ_i: the angle of rotation from the X_{i-1} axis to the X_i axis about the Z_{i-1} axis. It is the joint variable if joint i is rotary.

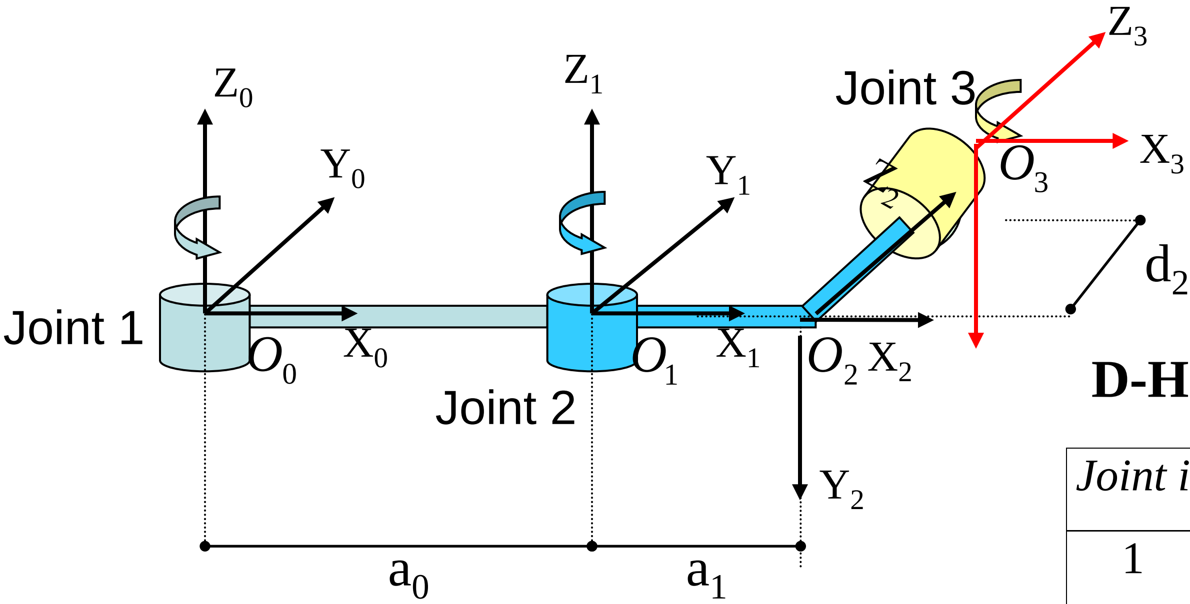
- *Joint distance* d_i : the distance from the origin of the (i-1) coordinate system to the intersection of the Z_{i-1} axis and the X_i axis along the Z_{i-1} axis. It is the joint variable if joint i is prismatic.

- *Link length* a_i : the distance from the intersection of the Z_{i-1} axis and the X_i axis to the origin of the ith coordinate system along the X_i axis.

- *Link twist angle* α_i: the angle of rotation from the Z_{i-1} axis to the Z_i axis about the X_i axis.

Robotics

Example I



D-H Link Parameter Table

Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_0
2	-90	a_1	0	θ_1
3	0	0	d_2	θ_2

α_i : rotation angle from Z_{i-1} to Z_i about X_i

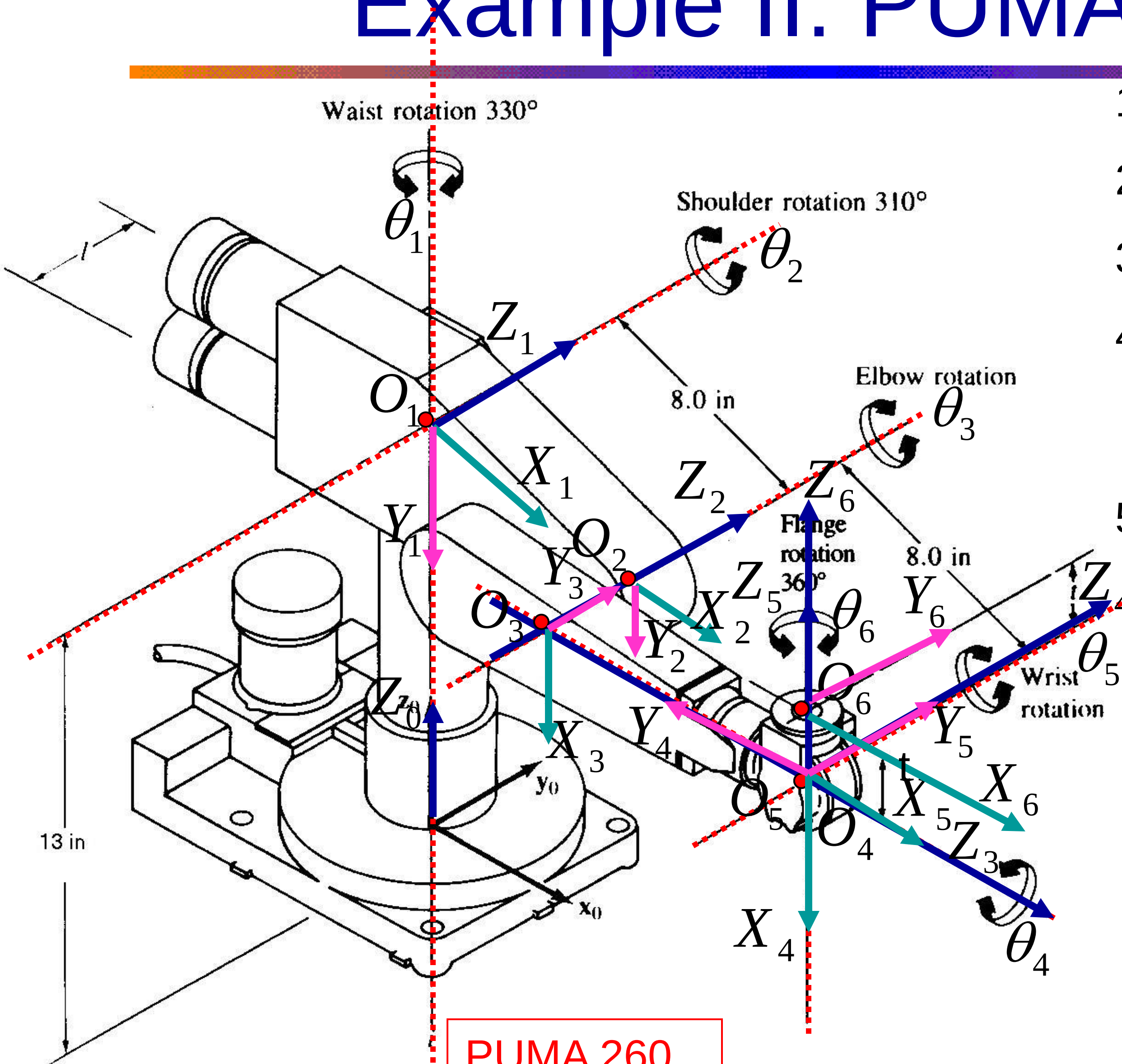
a_i : distance from intersection of Z_{i-1} & X_i to origin of i coordinate along X_i

d_i : distance from origin of (i-1) coordinate to intersection of Z_{i-1} & X_i along Z_{i-1}

θ_i : rotation angle from X_{i-1} to X_i about Z_{i-1}

Robotics

Example II: PUMA 260



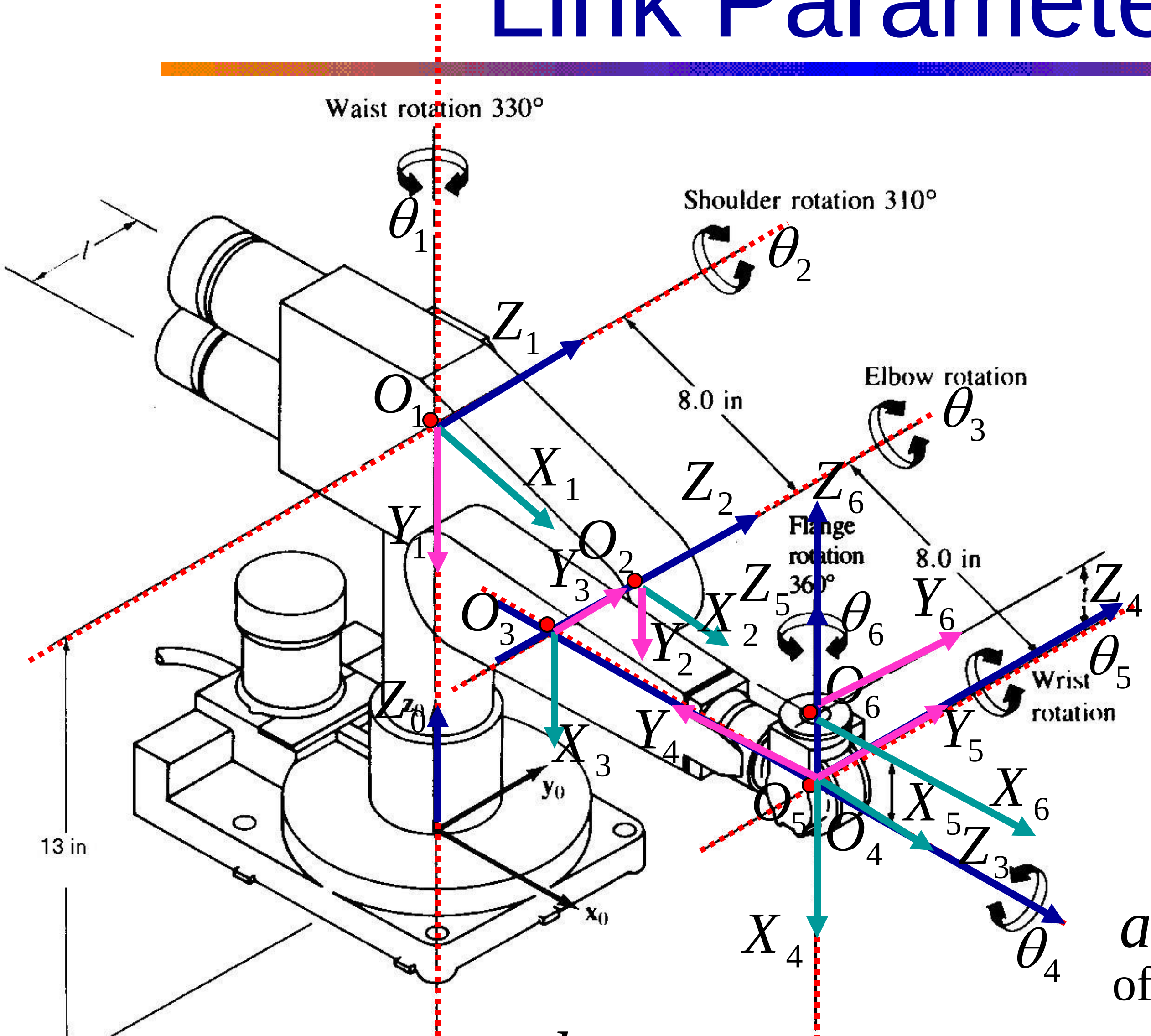
PUMA 260

1. Number the joints
2. Establish base frame
3. Establish joint axis Z_i
4. Locate origin, (intersect. of Z_i & Z_{i-1}) OR (intersect of common normal & Z_i)
5. Establish X_i, Y_i

$$X_i = \pm(Z_{i-1} \times Z_i) / \|Z_{i-1} \times Z_i\|$$

$$Y_i = +(Z_i \times X_i) / \|Z_i \times X_i\|$$

Link Parameters



J	θ_i	α_i	a_i	d_i
1	θ_1	-90	0	13
2	θ_2	0	8	0
3	θ_3	90	0	-l
4	θ_4	-90	0	8
5	θ_5	90	0	0
6	θ_6	0	0	t

θ_i : angle from X_{i-1} to X_i about Z_{i-1}

α_i : angle from Z_{i-1} to Z_i about X_i

a_i : distance from intersection of Z_{i-1} & X_i to O_i along X_i

Joint distance d_i : distance from O_{i-1} to intersection of Z_{i-1} & X_i along Z_{i-1}

Transformation between i-1 and i

- Four successive elementary transformations are required to relate the *i*-th coordinate frame to the (*i*-1)-th coordinate frame:
  - Rotate about the Z_{i-1} axis an angle of θ_i to align the X_{i-1} axis with the X_i axis.
  - Translate along the Z_{i-1} axis a distance of d_i, to bring X_{i-1} and X_i axes into coincidence.
  - Translate along the X_i axis a distance of a_i to bring the two origins O_{i-1} and O_i as well as the X axis into coincidence.
  - Rotate about the X_i axis an angle of α_i (in the right-handed sense), to bring the two coordinates into coincidence.

Robotics

Transformation between i-1 and i

- D-H transformation matrix for adjacent coordinate frames, *i* and *i-1.*
  - The position and orientation of the *i*-th frame coordinate can be expressed in the (*i-1*)th frame by the following homogeneous transformation matrix:

Source coordinate

Reference Coordinate

$$T_{i-1}^{i} = T(z_{i-1}, d_i)R(z_{i-1}, \theta_i)T(x_i, a_i)R(x_i, \alpha_i)$$

$$\begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robotics

Kinematic Equations

- Forward Kinematics $q = (q_1, q_2, \cdots q_n)$
  - Given joint variables
  - End-effector position & orientation $Y = (x, y, z, \phi, \theta, \psi)$
- Homogeneous matrix T_0^n
  - specifies the location of the ith coordinate frame w.r.t. the base coordinate system
  - chain product of successive coordinate transformation matrices of T_{i-1}^i

$$T_0^n = T_0^1 T_1^2 \ldots T_{n-1}^n$$

$$= \begin{bmatrix} R_0^n & P_0^n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} n & s & a & P_0^n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orientation matrix

Position vector

Robotics

Kinematics Equations

- Other representations
  - reference from, tool frame

$$T_{ref}^{tool} = B_{ref}^{0} T_0^n H_n^{tool}$$

  - Yaw-Pitch-Roll representation for orientation

$$T = R_{z,\phi} R_{y,\theta} R_{x,\psi}$$

$$= \begin{bmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representing forward kinematics

- Forward kinematics

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \Rightarrow \begin{bmatrix} p_x \\ p_y \\ p_z \\ \phi \\ \theta \\ \varphi \end{bmatrix}$$

- Transformation Matrix

$$T = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representing forward kinematics

- Yaw-Pitch-Roll representation for orientation

$$T_0^n = \begin{bmatrix} C\phi C\vartheta & C\phi S\theta S\psi - S\phi C\psi & C\phi S\theta C\psi + S\phi S\psi & p_x \\ S\phi C\theta & S\phi S\theta S\psi + C\phi C\psi & S\phi S\theta C\psi - C\phi S\psi & p_y \\ -S\theta & C\theta S\psi & C\theta C\psi & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^n = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

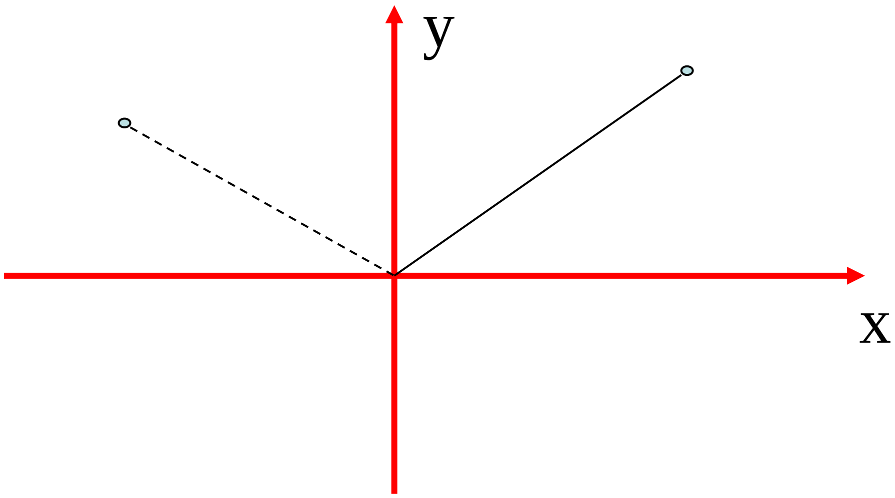
$$\theta = \sin^{-1}(-n_z)$$

$$\psi = \cos^{-1}\left(\frac{a_z}{\cos\theta}\right)$$

$$\phi = \cos^{-1}\left(\frac{n_x}{\cos\theta}\right)$$

Problem? Solution is inconsistent and ill-conditioned!!

atan2(y,x)



$$\theta = a\tan 2(y,x) = \begin{cases} 0^\circ \le \theta \le 90^\circ & for \quad +x \quad and \quad +y \\ 90^\circ \le \theta \le 180^\circ & for \quad -x \quad and \quad +y \\ -180^\circ \le \theta \le -90^\circ & for \quad -x \quad and \quad -y \\ -90^\circ \le \theta \le 0^\circ & for \quad +x \quad and \quad -y \end{cases}$$

Robotics

Yaw-Pitch-Roll Representation

$$T = R_{z,\phi} R_{y,\theta} R_{x,\psi}$$

$$= \begin{bmatrix} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & s_x & a_x & 0 \\ n_y & s_y & a_y & 0 \\ n_z & s_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Yaw-Pitch-Roll Representation

$$R_{z,\phi}^{-1}T = R_{y,\theta}R_{x,\psi}$$

$$\begin{bmatrix} C\phi & S\phi & 0 & 0 \\ -S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & s_x & a_x & 0 \\ n_y & s_y & a_y & 0 \\ n_z & s_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Equation A)

$$= \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\psi & -S\psi & 0 \\ 0 & S\psi & C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Yaw-Pitch-Roll Representation

- Compare LHS and RHS of Equation A, we have:

$$-\sin\phi\cdot n_x + \cos\phi\cdot n_y = 0 \quad\Rightarrow\quad \phi = a\tan 2(n_y, n_x)$$

$$\begin{cases}\cos\phi\cdot n_x + \sin\phi\cdot n_y = \cos\theta \\ n_z = -\sin\theta\end{cases} \quad\Rightarrow\quad \theta = a\tan 2(-n_z, \cos\phi\cdot n_z + \sin\phi\cdot n_y)$$

$$\begin{cases}-\sin\phi\cdot s_x + \cos\phi\cdot s_y = \cos\psi \\ -\sin\phi\cdot a_x + \cos\phi\cdot a_y = -\sin\psi\end{cases}$$

$$\Downarrow$$

$$\psi = a\tan 2(\sin\phi\cdot a_x - \cos\phi\cdot a_y, -\sin\phi\cdot s_x + \cos\phi\cdot s_y)$$

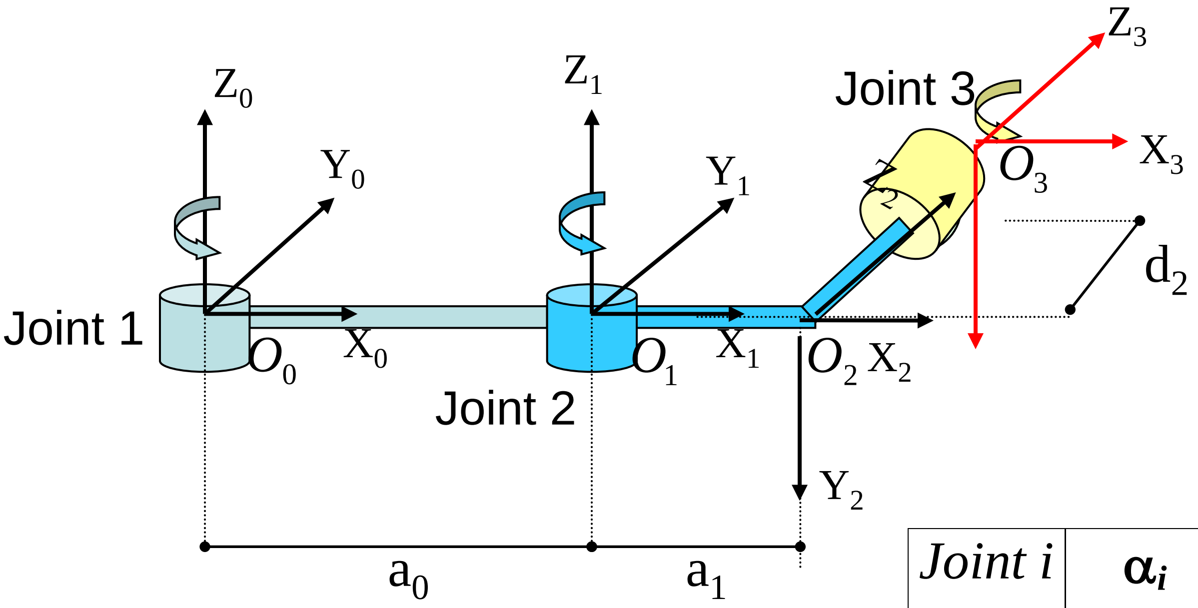
Robotics

Kinematic Model

- Steps to derive kinematics model:
  - Assign D-H coordinates frames
  - Find link parameters
  - Transformation matrices of adjacent joints
  - Calculate Kinematics Matrix
  - When necessary, Euler angle representation

Robotics

Example



Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_0
2	-90	a_1	0	θ_1
3	0	0	d_2	θ_2

Robotics

Example

Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_0
2	-90	a_1	0	θ_1
3	0	0	d_2	θ_2

$$T_{i-1}^{i} = \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

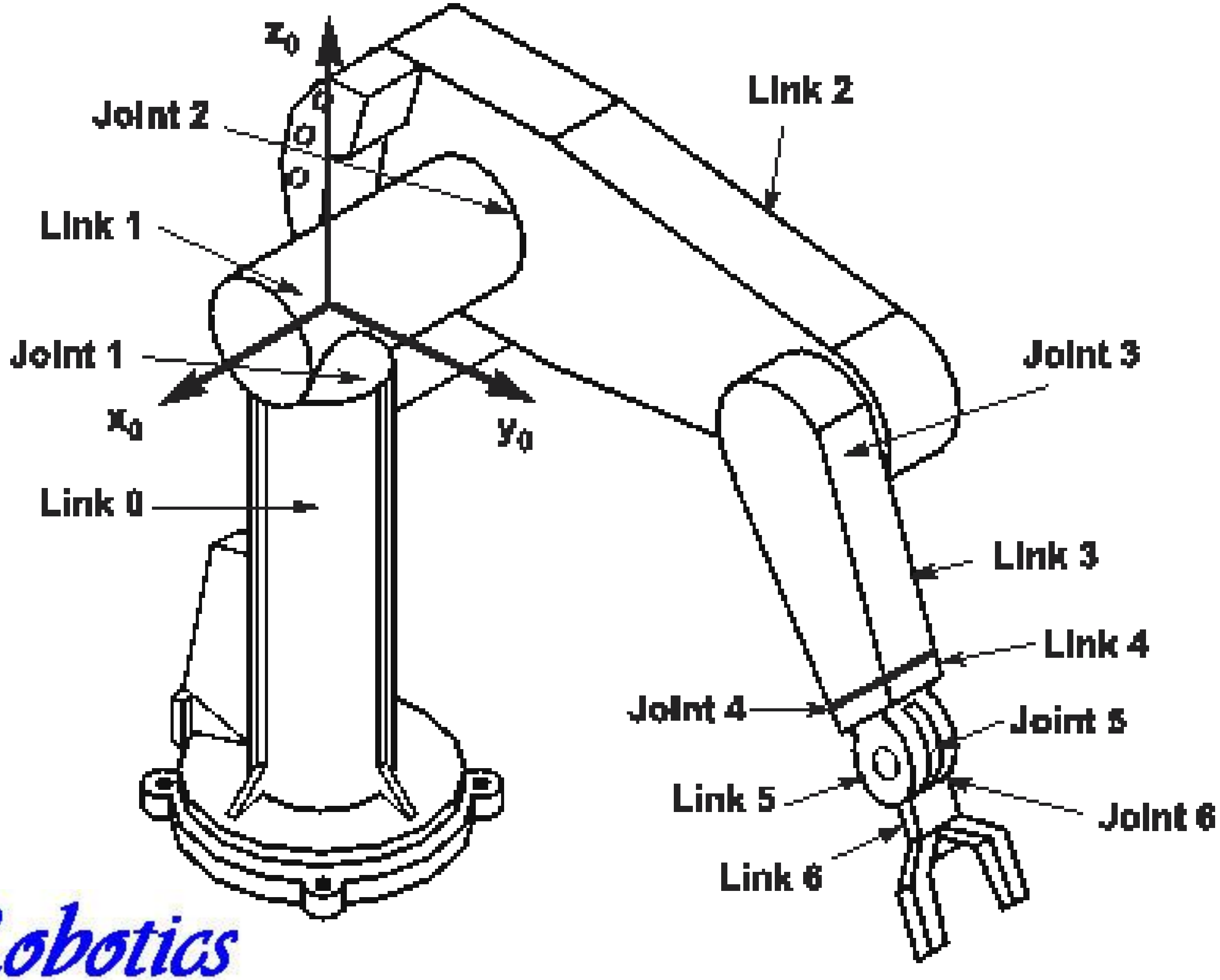
$$T_0^3 = (T_{\mathbf{0}}^1)(T_{\mathbf{1}}^2)(T_{\mathbf{2}}^3)$$

$$T_{\mathbf{0}}^1 = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & a_0\cos\theta_0 \\ \sin\theta_0 & \cos\theta_0 & 0 & a_0\sin\theta_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{\mathbf{1}}^2 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1\sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

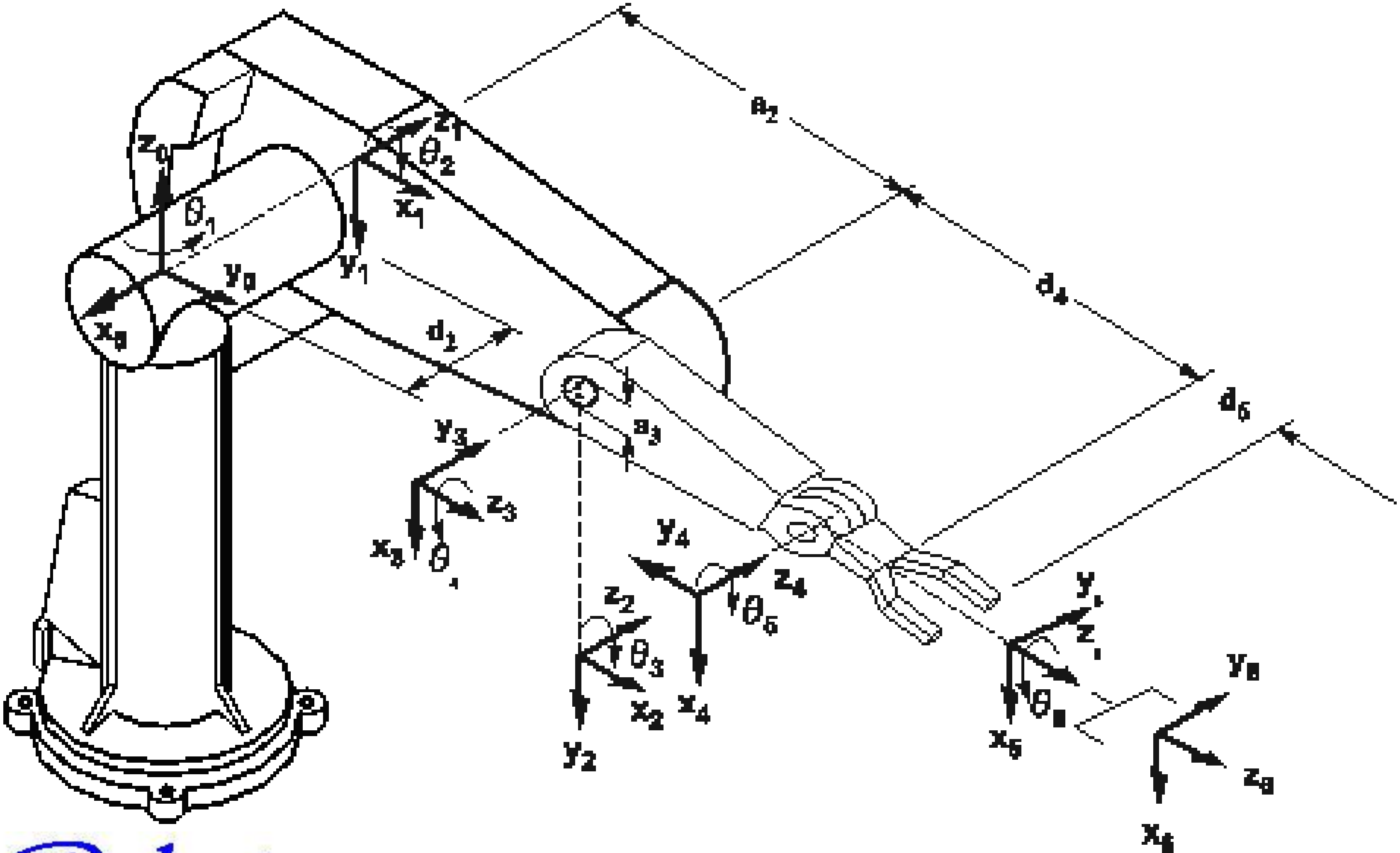
$$T_{\mathbf{2}}^3 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example: **Puma 560**



Robotics

Example: **Puma 560**



Robotics

Link Coordinate Parameters

PUMA 560 robot arm link coordinate parameters

Joint i	θ_i	α_i	a_i(mm)	d_i(mm)
1	θ_1	-90	0	0
2	θ_2	0	431.8	149.09
3	θ_3	90	-20.32	0
4	θ_4	-90	0	433.07
5	θ_5	90	0	0
6	θ_6	0	0	56.25

Robotics

Example: **Puma 560**

$$ {}^{0}T_{1} = \begin{pmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^{1}T_{2} = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}T_{3} = \begin{pmatrix} \cos\theta_3 & 0 & \sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & -\cos\theta_3 & a_3\sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^{3}T_{4} = \begin{pmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{4}T_{5} = \begin{pmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^{5}T_{6} = \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Robotics

Example: **Puma 560**

$$
{}^0\boldsymbol{T}_6 = {}^0\boldsymbol{T}_1\,{}^1\boldsymbol{T}_2\,{}^2\boldsymbol{T}_3\,{}^3\boldsymbol{T}_4\,{}^4\boldsymbol{T}_5\,{}^5\boldsymbol{T}_6 = \begin{pmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
\begin{aligned}
n_x &= c_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) - s_1(s_4c_5c_6 + c_4s_6) \\
n_y &= s_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) + c_1(s_4c_5c_6 + c_4s_6) \\
n_z &= -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6
\end{aligned}
$$

$$
\begin{aligned}
s_x &= c_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) - s_1(-s_4c_5s_6 + c_4c_6) \\
s_y &= s_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) + c_1(-s_4c_5s_6 + c_4c_6) \\
s_z &= s_{23}(c_4c_5s_6 + s_4c_6) - c_{23}s_5s_6
\end{aligned}
$$

$$
\begin{aligned}
a_x &= c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5 \\
a_y &= s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5 \\
a_z &= -s_{23}c_4s_5 + c_{23}c_5)
\end{aligned}
$$

$$
\begin{aligned}
p_x &= c_1(d_6(c_{23}c_4s_5 + s_{23}c_5) + s_{23}d_4 + a_3c_{23} + a_2c_2) - s_1(d_6s_4s_5 + d_2) \\
p_y &= s_1(d_6(c_{23}c_4s_5 + s_{23}c_5) + s_{23}d_4 + a_3c_{23} + a_2c_2) + c_1(d_6s_4s_5 + d_2) \\
p_z &= d_6(c_{23}c_5 - s_{23}c_4s_5) + c_{23}d_4 - a_3s_{23} - a_2s_2
\end{aligned}
$$

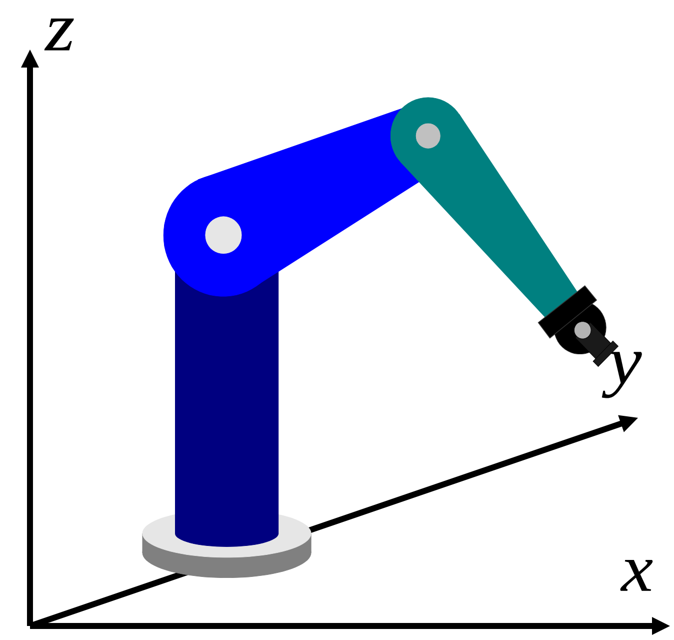
Robotics

Inverse Kinematics

- Given a desired position (P) & orientation (R) of the end-effector

$$q = (q_1, q_2, \cdots q_n)$$

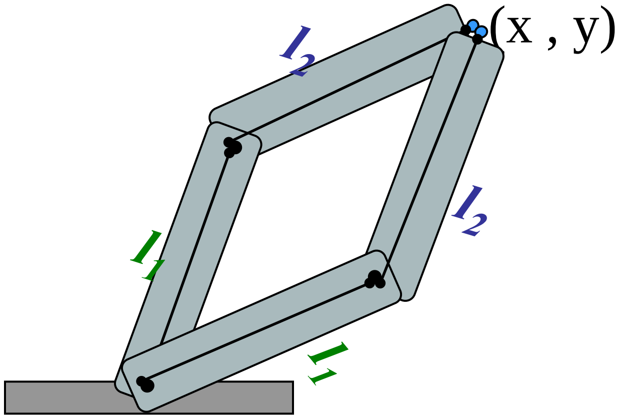
- Find the joint variables which can bring the robot the desired configuration



Robotics

Inverse Kinematics

- More difficult
  - Systematic closed-form solution in general is not available
  - Solution not unique
    - Redundant robot
    - Elbow-up/elbow-down configuration
  - Robot dependent



Robotics

Inverse Kinematics

- Transformation Matrix

$$T=\begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5 T_5^6 \Rightarrow \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}$$

Special cases make the closed-form arm solution possible:

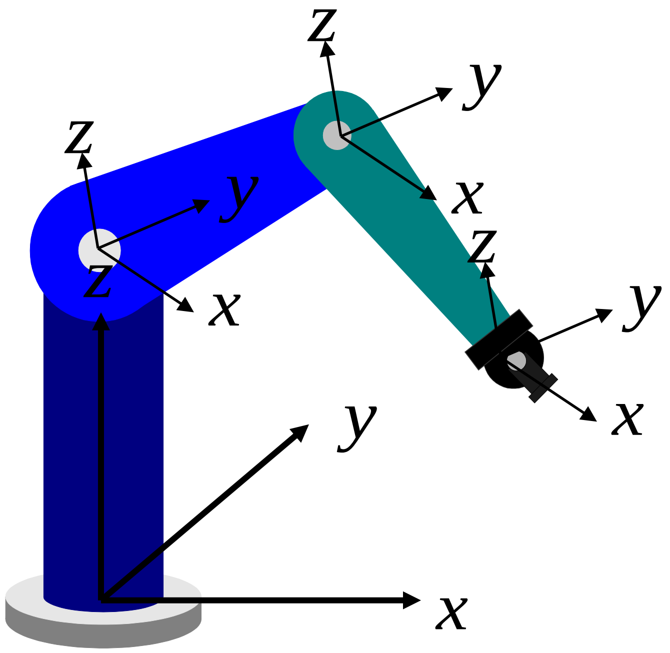
1. Three adjacent joint axes intersecting (PUMA, Stanford)
2. Three adjacent joint axes parallel to one another (MINIMOVER)

Robotics

Thank you!

Homework

posted on my web site.



Robotics