4.1 Predicate calculus *First order predicate logic, FOL*, the simplest form of predicate logic.
Here is its BNF:

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Sentence → AtomicSentence
        | Sentence Connective Sentence
        | Quantifier Variable, . . . Sentence
        | ¬Sentence
        | (Sentence)
AtomicSentence → Predicate(Term, . . .) | Term = Term

Term → Function(Term, . . .) | Constant | Variable
Connective → ∨ | ∧ | ⇒ | ⇔
Quantifier → ∀ | ∃
Constant → A | X | John | . . .
Variable → a | x | s| john | . . .
Predicate→ Before | HasColor | Raining | . . .
Function → Mother | LeftLegOf | . . .
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Universal quantification (∀) $\forall x\ Cat(x) \Rightarrow Mammal(x)$

Existential quantification (∃) $\exists x\ Sister(x, Spot) \wedge Cat(x)$

A term with no variables is called a **ground term.**

Connections between ∀ and ∃ (De Morgan rules)

$$\forall x\ \neg P \equiv \neg\exists x\ P \qquad \neg P \wedge \neg Q \equiv \neg(P \vee Q)$$
$$\neg\forall x\ P \equiv \exists x\ \neg P \qquad \neg(P \wedge Q) \equiv \neg P \vee \neg Q$$
$$\forall x\ P \equiv \neg\exists x\ \neg P \qquad P \wedge Q \equiv \neg(\neg P \vee \neg Q)$$
$$\exists x\ P \equiv \neg\forall x\ \neg P \qquad P \vee Q \equiv \neg(\neg P \wedge \neg Q)$$

4.2 Knowledge representation using FOL

Sentences that are in the knowledge base initially are called "**axioms**".
Theorems are entailed by the axioms.
An axiom of form $\forall x, y\ P(x, y) \Leftrightarrow$ ... is often called a **definition** of *P*.

Example (1): The kinship domain	**Example (2): The domain of sets**
One's mother is one's female parent: $\forall\ m,c\ Mother(m,c) \Leftrightarrow Female(m) \wedge Parent(m,c)$ One's husband is one's male spouse: $\forall w,h\ Husband(h,w) \Leftrightarrow Male(h) \wedge Spouse(h,w)$ Male and female are disjoint categories: $\forall\ x\ Male(x) \Leftrightarrow \neg Female(x)$ Parent and child are inverse relations: $\forall\ p, c\ Parent(p, c) \Leftrightarrow Child(c, p)$ A grandparent is a parent of one's parent: $\forall g,c\ \ Grandparent(g,c) \Leftrightarrow \exists p\ \ Parent(g,p) \wedge Parent(p,c)$ A sibling is another child of one's parents: $\forall\ x, y\ Sibling(x, y) \Leftrightarrow \exists\ p\ Parent(p, x) \wedge Parent(p, y)$	*1.* Empty set has no elements adjoined into it. $\neg\exists\ x, s\ \{x \mid s\} = \{\}$ 2. Adjoining an element in the set has no effect: $\forall\ x, s\ \ x \in s \Leftrightarrow s = \{x \mid s\}$ *3.* A set is a subset of another if and only if all first's members are members of the second. $\forall s_1, s_2\ \ s_1 \subseteq s_2 \Leftrightarrow (\forall\ x\ \ x \in s_1 \Rightarrow x \in s_2)$ *4.* Two sets are equal if and only if each is a subset of the other. $\forall\ s_1, s_2\ (s_1 = s_2) \Leftrightarrow (\ s_1 \subseteq s_2 \wedge s_2 \subseteq s_1\ )$ 5. An object is a member of the intersection of two sets if and only if it is a member of each of the sets. $\forall x, s_1, s_2\ \ x \in (s_1 \cap s_2)) \Leftrightarrow (\ x \in s_1 \wedge x \in s_2)$ 6. An object is a member of the union of two sets if and only if it is a member of either set. $\forall x, s_1, s_2\ \ x \in (s_1 \cup s_2)) \Leftrightarrow (\ x \in s_1 \vee x \in s_2)$