Consider next the argument:

$$P \leftrightarrow Q$$
$$(S \vee T) \rightarrow Q$$
$$\underline{\neg P \vee (\neg T \,\&\, R)}$$
$$T \rightarrow U$$

This argument has 6 distinct statement letters, and hence a truth table for it requires 64 rows. Derivation of the conclusion of the premises using our inference and replacement rules is :

1. P ↔ Q	Premise
2. (S v T) → Q	Premise
3. ¬P v (¬T & R)	Premise
4. (P → Q) & (Q → P)	1 Equiv
5. Q → P	4 Simp
6. (S v T) → P	2,5 HS
7. P → (¬T & R)	3 Impl
8. (S v T) → (¬T & R)	6,7 HS
9. ¬(S v T) v (¬T & R)	8 Impl
10. (¬S & ¬T) v (¬T & R)	9 DM
11. [(¬S & ¬T) v ¬T] & [(¬S & ¬T) v R]	10 Dist
12. (¬S & ¬T) v ¬T	11 Simp
13. ¬T v (¬S & ¬T)	12 Com
14. (¬T v ¬S) & (¬T v ¬T)	13 Dist
15. ¬T v ¬T	14 Simp
16. ¬T	15 Taut
17. ¬T v U	16 Add
18. T → U	17 Impl

3.2.5 Conditional and Indirect Proofs

A conditional proof is a derivation technique used to establish a conditional wff, i.e., a wff whose main operator is the sign '→'. This is done by constructing a sub-derivation within a derivation in which the antecedent of the conditional is assumed as a hypothesis. If, by using the inference rules and rules of derivation (and possibly additional sub-derivations), it is possible to arrive at the consequent, it is permissible to end the sub-derivation and conclude the truth of the conditional statement within the main derivation, citing the sub-derivation as a conditional proof, or 'CP'.

Consider the following example argument:

$$P \rightarrow (Q \vee R)$$
$$P \rightarrow \neg S$$
$$\underline{S \leftrightarrow Q}$$
$$P \rightarrow R$$

It is easier to establish validity of using a conditional derivation.

1. P → (Q v R)	Premise
2. P → ¬S	Premise
3. S ↔ Q	Premise
4. P	Assumption
5. Q v R	1,4 MP
6. ¬S	2,4 MP
7. (S → Q) & (Q → S)	3 Equiv
8. Q → S	7 Simp
9. ¬Q	6,8 MT

10. R	5,9 DS
11. P → R	4-10 CP

Here in order to establish the conditional statement "P → R", we constructed a sub-derivation, which is the indented portion found at lines 4-10. First, we assumed the truth of 'P', and found that with it, we could derive 'R'. Given the premises, we therefore had shown that if 'P' were also true, so would be 'R'. Therefore, on the basis of the sub-derivation we were justified in concluding "P → R".

Another common method is that of *indirect proof*, also known as proof by *reduction ad absurdum*. In an *indirect proof (IP),* our goal is to demonstrate that a certain wff is false on the basis of the premises. We begin by assuming the opposite of that which we're trying to prove.

If we can demonstrate an obvious contradiction, i.e., a statement of the form α & ¬α, we can conclude that the assumed statement must be false, because anything that leads to a contradiction must be false. For example, consider the argument:

P → Q
P → (Q → ¬P)
¬P

It is easier to prove that "¬P" is true by showing that, given the premises, it would be impossible for 'P' to be true by assuming that it is and showing this to be absurd.

1. P → Q	Premise
2. P → (Q → ¬P)	Premise
3. P	Assumption
4. Q	1,3 MP
5. Q → ¬P	2,3 MP
6. ¬P	4,5 MP
7. P & ¬P	3,6 Conj
8. ¬P	3-7 IP

Here we were attempting to show that "¬P" was true given the premises. To do this we assumed instead that 'P' was true. Since this assumption was impossible, we were justified in concluding that 'P' is false, i.e., that "¬P" is true.

There are many equivalent systems of natural deduction, all coinciding relatively closely to ordinary reasoning patterns.

One disadvantage this method has, however, is that, unlike truth tables, it does not provide a means for recognizing that an argument is invalid. If an argument is invalid, there is no deduction for it in the system.