3.1 Logic and Knowledge

knowledge base (KB) is a set of **sentences** expressed in a language called a **knowledge representation language. Inference** (deriving new sentences from old).

A system of *natural deduction* , derived by Copi consists in :

Rules of Inference

Modus ponens (MP): $\alpha \to \beta$, α / β	**Modus tollens (MT):** $\alpha \to \beta$, $\neg\beta$ / $\neg\alpha$	**Hypothetical syllogism (HS):** α → β, β → γ / α → γ	**Conjunction (Conj):** α, β / α & β	**Absorption (Abs):** α → β / α → (α & β)
Addition (Add): $\frac{\alpha}{\alpha \vee \beta}$ $\frac{\beta}{\alpha \vee \beta}$	**Simplification (Simp):** $\frac{\alpha \& \beta}{\alpha}$ $\frac{\alpha \& \beta}{\beta}$	**Disjunctive syllogism (DS):** $\frac{\alpha \vee \beta}{\neg\alpha}$ $\frac{\alpha \vee \beta}{\neg\beta}$		**Constructive dilemma (CD):** (α → γ) & (β → δ) / α v β / γ v δ

Rules of Replacement

Double negation (DN):	**DeMorgan's Laws (DM):** (two forms)
¬¬α is replaceable with α	¬(α & β) is replaceable with ¬α v ¬β
Commutativity (Com): (two forms)	¬(α v β) is replaceable with ¬α & ¬β
α & β is replaceable with β & α	**Material Implication (Impl):**
α v β is replaceable with β v α	α → β is replaceable with ¬α v β
Associativity (Assoc): (two forms)	**Transposition (Trans):**
(α & β) & γ is replaceable with α& (β & γ)	α → β is replaceable with ¬β → ¬α
(α v β) v γ is replaceable with α v (β v γ)	**Distribution (Dist):** (two forms)
Tautology (Taut): (two forms)	α&(βvγ) is replaceable with (α&β)v(α&γ)
α is replaceable with α & α	αv(β&γ) is replaceable with (αvβ)&(αvγ)
α is replaceable with α v α	**Material Equivalence (Equiv):** (2 forms)
Exportation (Exp):	α ↔ β is replaceable with (α→β)&(β → α)
α→(β→γ) is replaceable with (α&β)→γ	α ↔ β is replaceable with (α&β) v(¬α &¬β)

Consider the argument in Natural Language, followed with the chain of reasoning:

Either cat fur or dog fur was found at the scene of the crime. If dog fur was found at the scene of the crime, Officer had an allergy attack. If cat fur was found at the scene of the crime, then Macavity is responsibile for the crime. But Officer didn't have an allergy attack, and so therefore Macavity must be responsible for the crime.

1. *Either cat fur was found at the scene of the crime, or dog fur was found at the scene of the crime.* (Premise)
2. *If dog fur was found at the scene of the crime, then Officer had an allergy attack.* (Premise)
3. *If cat fur was found at the scene of the crime, then Macavity is responsible for the crime.* (Premise)
4. *Officer did not have an allergy attack.* (Premise)
5. *Dog fur was not found at the scene of the crime.*(from 2,4)
6. *Cat fur was found at the scene of the crime.* (from 1 and 5)
7. *Macavity is responsible for the crime.*(Conclusion from 3,6)

1. C v D	Premise
2. C → O	Premise
3. D → M	Premise
4. ¬O	Premise
5. ¬C	2,4 MT
6. D	1,5 DS
7. M	2,6 MP

1. C v D	Premise
2. C → O	Premise
3. D → M	Premise
4. ¬O	Premise
5. (C→O)&(D→M)	2,3 Conj
6. O v M	1,5 CD
7. M	4,6 DS