2.5 Tautologies, Logical Equivalence and Validity

Definition: a wff is a *tautology* if and only if it is true for all possible truth-value assignments to the statement letters making it.

P	P	v	¬	P
T	T	T	F	T
F	F	T	T	F

Definition: a wff is a *self-contradiction* if and only if it is false for all possible truth-value assignments to statement letters making it.

P	P	&	¬	P
T	T	F	F	T
F	F	F	T	F

P	Q	(P	→	Q)	&	(P	→	¬	Q)
T	T	T	T	T	F	T	F	F	T
T	F	T	F	F	F	T	T	T	F
F	T	F	T	T	T	F	T	F	T
F	F	F	T	F	T	F	T	T	F

A statement that is neither self-contradictory nor tautological is called a *contingent* statement.

P	Q	P	v	Q	¬	(P	↔	¬	Q)
T	T	T	T	T	T	T	F	F	T
T	F	T	T	F	F	T	T	T	F
F	T	F	T	T	F	F	T	F	T
F	F	F	F	F	T	F	F	T	F

Definition: two wffs are *consistent* if and only if there is at least one possible truth-value assignment to the statement letters making them up that makes both wffs true.

Definition: two wffs are *inconsistent* if and only if there is no truth-value assignment that makes them both true.

P	Q	(P	→	Q)	&	P	¬	(Q	v	¬	P))
T	T	T	T	T	T	T	F	T	T	F	T
T	F	T	F	F	F	T	T	F	F	F	T
F	T	F	T	T	F	F	F	T	T	T	F
F	F	F	T	F	F	F	F	F	T	T	F

Definition: two statements are *logically equivalent* if and only if all truth-value assignments are the same for the whole statements.

P	Q	¬	P	→	¬	Q	¬	(Q	&	¬	P))
T	T	F	T	T	F	T	T	T	F	F	T
T	F	F	T	T	T	F	T	F	F	F	T
F	T	T	F	F	F	T	F	T	T	T	F
F	F	T	F	T	T	F	T	F	F	T	F

Definition: a wff β is said to be a *logical consequence* of a set of wffs $\alpha_1, \alpha_2, ..., \alpha_n$, if and only if there is no truth-value assignment to the statement letters making up these wffs that makes all of $\alpha_1, \alpha_2, ..., \alpha_n$ true but does not make β true.

An argument is *logically valid* if and only if its conclusion is a logical consequence of its premises. For example, consider the following:

$$P \rightarrow Q$$
$$\underline{\neg Q \rightarrow P}$$
$$Q$$

P	Q	P	→	Q	¬	Q	→	P	Q
T	T	T	T	T	F	T	T	T	T
T	F	T	F	F	T	F	T	T	F
F	T	F	T	T	F	T	T	F	T
F	F	F	T	F	T	F	F	F	F