

2.1 Statement of **proposition** is a declarative sentence that is true or false, but not both.

**Examples of Statements of Proposition**

- o  $2+3=5$ : statement that is true
- o *Do you speak English?*? This is a question, not a statement
- o  $3-x=5$ : is a declarative sentence, but not a statement since it depends on the value of x
- o *Take two aspirins*: is a command, not a statement

2.2 Syntax and Formation Rules of PL (Propositional logic)

Connective	Description
$\wedge$ (and)	Such as $P \wedge Q$ is called a <b>conjunction</b> ; its parts are <b>conjuncts</b> . Some logical books use the sign '&' for conjunction.
$\vee$ (or)	Such as $A \vee (P \wedge Q)$ , is a <b>disjunction</b> of the <b>disjuncts</b> A and $(P \wedge Q)$ . The <i>exclusive</i> sense of 'or' is symbolized as ' $\oplus$ ' or as ' <u>v</u> '.
$\Rightarrow$ (implies)	Such as $(P \wedge Q) \Rightarrow R$ is called an <b>implication</b> (conditional, <b>rules or if-then</b> ). Its <b>premise or antecedent</b> is $P \wedge Q$ , and its <b>conclusion or consequent</b> is R. This symbol is sometimes written as $\rightarrow$ . The <b>converse</b> of $p \Rightarrow q$ is the implication that $q \Rightarrow p$ The <b>contrapositive</b> of $p \Rightarrow q$ is the implication that $\sim q \Rightarrow \sim p$
$\Leftrightarrow$ (equivalent)	The sentence $(P \vee Q) \Leftrightarrow (Q \vee P)$ is an <b>equivalence (biconditional)</b> . This symbol is sometimes written as $\leftrightarrow$ or as $\equiv$
$\neg$ (not)	Such as $\neg P$ is called <b>negation</b> of P; $\neg$ is the only unary connective The sign '~' is sometimes used in place of ' $\neg$ '.

Here is the **BNF**, a formal method for describing syntax, of PL.

<i>Sentence</i>	$\rightarrow$	<i>AtomicSentence</i>   <i>ComplexSentence</i>
<i>AtomicSentence</i>	$\rightarrow$	<b>True</b>   <b>False</b>   <i>Symbol</i>
<i>Symbol</i>	$\rightarrow$	<b>P</b>   <b>Q</b>   <b>R</b>   ...
<i>ComplexSentence</i>	$\rightarrow$	$\neg$ <i>Sentence</i>
		( <i>Sentence</i> $\wedge$ <i>Sentence</i> )
		( <i>Sentence</i> $\vee$ <i>Sentence</i> )
		( <i>Sentence</i> $\Rightarrow$ <i>Sentence</i> )
		( <i>Sentence</i> $\Leftrightarrow$ <i>Sentence</i> )

The order of precedence in propositional logic is (from highest to lowest):  $\neg$ ,  $\wedge$ ,  $\vee$ ,  $\Rightarrow$ , and  $\Leftrightarrow$ .

Hence, the sentence  $\neg P \vee Q \wedge R \Rightarrow S$  is equivalent to the sentence  $((\neg P) \vee (Q \wedge R)) \Rightarrow S$ .

2.3 Truth Functions and Truth Tables

$\alpha$	$\beta$	$\alpha \& \beta$	$\alpha \vee \beta$	$\alpha \oplus \beta$	$\alpha \rightarrow \beta$	$\alpha \leftrightarrow \beta$
T	T	T	T	F	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T

$\alpha$	$\neg \alpha$
T	F
F	T

P	Q	R		((P	&	Q)	$\rightarrow$	$\neg$	R)
T	T	T		T	T	T	F	F	T
T	T	F		T	T	T	T	T	F
T	F	T		T	F	F	T	F	T
T	F	F		T	F	F	T	T	F
F	T	T		F	F	T	T	F	T
F	T	F		F	F	T	T	T	F
F	F	T		F	F	F	T	F	T
F	F	F		F	F	F	T	T	F

A wff (well-formed formula) has n distinct letters, the number of truth-value assignments is  $2^n$ .