1.1 Why Mathematical Logic?

Science is *communicable* knowledge. Language is naturally *ambiguous*, so to achieve precision, we have to use Mathematical Logic (ML). MLs are formal languages with :

Syntax- defines the sentences in the language

Semantics - define the "meaning" of sentences;

Example: the language of arithmetic

Syntax of this language says: $x+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence

Semantics of this language says: $x+2 \geq y$ is true in a world where $x = 7$, $y = 1$

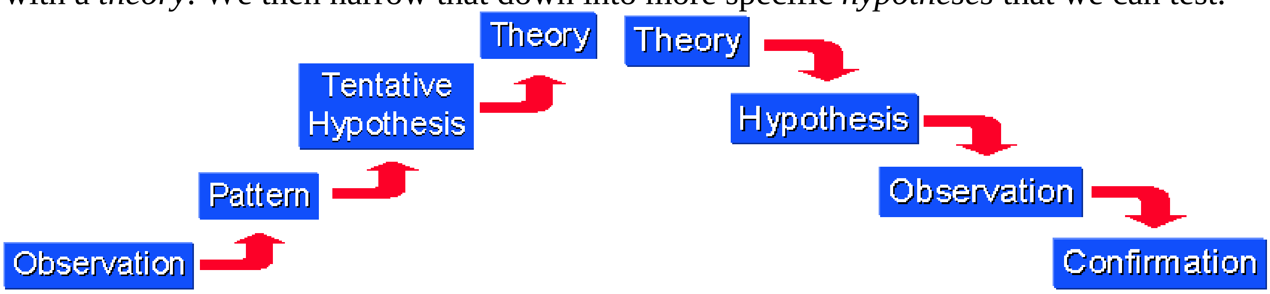
Certain Language	**Ontology**	**Epistemology**
Propositional logic	Facts	True/False
First-order logic (FOL)	Facts, objects, relations	True/False
Temporal logic	Facts,objects,relations,time	True/False
Uncertainty Language		
Probability	Facts	Degree of belief [0,1]
Fuzzy logic	Facts	Degree of truth [0,1]

Ontology: what exists in the world, **epistemology**: what the agent knows.

FOL is the only logic that is both consistent and complete as proved by Hilbert

1.2 Deductive and Inductive Thinking

Inductive reasoning (bottom up) works from specific observations to generalizations & theories. Deductive reasoning (top-down) works from the more general to the more specific. We begin with a *theory*. We then narrow that down into more specific *hypotheses* that we can test.



1.3 Induction and Recursion

The recursion logic justifies representing $f(n+1)$ in terms of $f(n)$.

Example: Show by mathematical induction that for all $n \geq 1$, $1+2+3+\ldots+n = \frac{n(n+1)}{2}$

Solution :

Let $P(n) : 1+2+3+\ldots+n = \frac{n(n+1)}{2}$

BASIS STEP

$P(1)$ is the statement : $1 = \frac{1(1+1)}{2}$ which is true.

INDUCTIVE STEP

Assume at $k \geq 1$, prove the truth of $P(k+1)$:

$1+2+3+\ldots+k = \frac{k(k+1)}{2}$ .... (1)

$1+2+3+\ldots+(k+1) = \frac{(k+1)((k+1)+1)}{2}$

LHS = $\underline{(1+2+3+\ldots+k)} + (k+1) =$

$= (k+1)\left(\frac{k}{2}+1\right)$ using (1)

$= \frac{(k+1)((k+1)+1)}{2}$ = RHS

1.4 Argument Structure

①[Today is either Tuesday or Wednesday.] But ②[it can't be Wednesday,] (since) ③[the doctor's office was open this morning,] and ④[that office is always closed on Wednesday.] (Therefore,) ⑤[today must be Tuesday.]