Sheet 2

8) Given the universe of discourse = {1,2,3,4}, express each of the following using logical connectives ($\neg$, $\wedge$, $\vee$) but without using quantifiers.

a. $\forall x\, P(x)$
b. $\exists x\, P(x)$
c. $\forall x\, ((x < 3) \rightarrow P(x))$
d. $\exists x\, ((x \text{ is even}) \wedge \neg P(x))$

9) Use quantifiers to express the following propositions

a. Goldbach's conjecture: Every integer greater than 2 is the sum of two primes.
b. Fermat's last theorem: "… it is impossible to write a cube as the sum of two cubes, a fourth power as the sum of two fourth powers and in general any power beyond the second as the sum of two similar powers".

Answer of sheet 2

8) Given the universe of discourse = {1,2,3,4}, express each of the following using logical connectives ($\neg$, $\wedge$, $\vee$) but without using quantifiers.

a. $\forall x\, P(x)$ $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$
b. $\exists x\, P(x)$ $P(1) \vee P(2) \vee P(3) \vee P(4)$
c. $\forall x\, ((x < 3) \rightarrow P(x))$ $P(1) \wedge P(2)$
d. $\exists x\, ((x \text{ is even}) \wedge \neg P(x))$ $\neg P(2) \vee \neg P(4)$

9) Use quantifiers to express the following propositions

a. Goldbach's conjecture: Every integer greater than 2 is the sum of two primes.

$$\forall x \in Z\ (x > 2 \rightarrow \exists p\ \exists q\ Prime(p) \wedge Prime(q) \wedge x = p+q)$$

b. Fermat's last theorem: "… it is impossible to write a cube as the sum of two cubes, a fourth power as the sum of two fourth powers and in general any power beyond the second as the sum of two similar powers".

$$\forall w \in Z\ (w>2 \rightarrow \forall x \in Z\ \forall y \in Z\ \forall z \in Z\ (\neg (x^w = y^w + z^w)))$$