[1] Write a BNF for FOL.
Then show , with examples , the differences between
i- FOL notation and Prolog
ii- FOL notation and Lisp

$$
\begin{aligned}
\mathit{Sentence} &\rightarrow \mathit{AtomicSentence} \\
&\quad |\ \mathit{Sentence\ Connective\ Sentence} \\
&\quad |\ \mathit{Quantifier\ Variable}, \ldots\ \mathit{Sentence} \\
&\quad |\ \neg \mathit{Sentence} \\
&\quad |\ (\mathit{Sentence}) \\
\\
\mathit{AtomicSentence} &\rightarrow \mathit{Predicate}(\mathit{Term}, \ldots) \mid \mathit{Term} = \mathit{Term} \\
\\
\mathit{Term} &\rightarrow \mathit{Function}(\mathit{Term}, \ldots) \mid \mathit{Constant} \mid \mathit{Variable} \\
\\
\mathit{Connective} &\rightarrow \vee \mid \wedge \mid \Rightarrow \mid \Leftrightarrow \\
\mathit{Quantifier} &\rightarrow \forall \mid \exists \\
\mathit{Constant} &\rightarrow \mathrm{A} \mid \mathrm{X} \mid \mathrm{John} \mid \ldots \\
\mathit{Variable} &\rightarrow a \mid x \mid s \mid \mathit{john} \mid \ldots \\
\mathit{Predicate} &\rightarrow \mathit{Before} \mid \mathit{HasColor} \mid \mathit{Raining} \mid \ldots \\
\mathit{Function} &\rightarrow \mathit{Mother} \mid \mathit{LeftLegOf} \mid \ldots
\end{aligned}
$$

i- Prolog

- ✓ It uses uppercase letters for **variables** and lowercase for constants.
- ✓ It also reverses the order of **implications**, writing Q :- P instead of $P \Rightarrow Q$.
- ✓ A comma is used both to separate arguments and for **conjunction**,
- ✓ and a period marks the end of a sentence:

example:

```
cat(X)  :- furry(X), meows(X),  has(X,claws).
```

ii- Lisp

- ✓ a consistent **prefix** notation is common.
- ✓ Each sentence and non-constant term is surrounded by parentheses,
- ✓ and the **connectives** come first, just like the predicate and function symbols.
- ✓ **variables** are usually distinguished by an initial ? or $ character (Because Lisp does not distinguish between uppercase and lowercase symbols)

example:

```
(forall ?x
  (implies (and (furry ?x) (meows ?x) (has ?x claws))
           (cat ?x)))
```

[2] In the kinship domain, represent each of the following sentence into FOL

i. One's mother is one's female parent
ii. One's husband is one's male spouse
iii. Male and female are disjoint categories
iv. Parent and child are inverse relations
v. A grandparent is a parent of one's parent
vi. A sibling is another child of one's parents

One's mother is one's female parent:

$$\forall\, m, c\ Mother(c) = m \Leftrightarrow Female(m) \wedge Parent(m, c)$$

One's husband is one's male spouse:

$$\forall w, h\ Husband(h, w) \Leftrightarrow Male(h) \wedge Spouse(h, w)$$

Male and female are disjoint categories:

$$\forall\, x\ Male(x) \Leftrightarrow \neg Female(x)$$

Parent and child are inverse relations:

$$\forall\, p, c\ Parent(p, c) \Leftrightarrow Child(c, p)$$

A grandparent is a parent of one's parent:

$$\forall g, c\ Grandparent(g, c) \Leftrightarrow \exists\, p\ Parent(g, p) \wedge Parent(p, c)$$

A sibling is another child of one's parents:

$$\forall\, x, y\ Sibling(x, y) \Leftrightarrow x \neq y \wedge \exists\, p\ Parent\ (p, x) \wedge Parent\ (p, y)$$

[3] give the complete FOL definition for set theory

Hint : use the normal vocabulary of set theory:

EmptySet is a constant written ϕ ,
Member and *Subset* are predicates written x $\in$ s1 , s1 $\subseteq$ s2
Intersection, Union, and *Adjoin* are functions written s1 $\cap$ s2 , s1 $\cup$ s2 , {x | s}.
Set is a predicate that is true only of sets.

1. The only sets are the empty set and those made by adjoining something to a set.

$$\forall s\ Set(s) \Leftrightarrow (s = \phi) \vee (\exists\, x, s_2\ Set(s_2) \wedge s = \{\, x |\ s2\}\,)$$

2. The empty set has no elements adjoined into it. (In other words, there is no way to decompose *EmptySet* into a smaller set and an element.)

$$\neg \exists\ \ x, s\ \{x |\ s\} = \phi$$

3. Adjoining an element already in the set has no effect:

$$\forall\, x, s\ \ x \in s \Leftrightarrow s = \{\, x |\ s\}$$

4. The only members of a set are the elements that were adjoined into it. We express this recursively, saying that *x* is a member of *s* if and only if s is equal to some set s_2 adjoined with some element *y*, where either *y* is the same as *x* or *x* is a member of s_2.

$$\forall\, x, s\ \ x \in s \Leftrightarrow [\, \exists\, y, s_2\ (s = \{\, y |\ s_2\}\ \wedge (x = y \vee x \in s_2))]$$

5. A set is a subset of another if and only if all of the first set's members are members of the second set.

$$\forall s_1, s_2\ \ s_1 \subseteq s_2 \Leftrightarrow (\forall\, x\ \ x \in s_1 \Rightarrow x \in s_2)$$

6. Two sets are equal if and only if each is a subset of the other.

$$\forall\, s_1, s_2\ (s_1 = s_2) \Leftrightarrow (\, s_1 \subseteq s_2 \wedge s_2 \subseteq s_1\,)$$

7. An object is a member of the intersection of two sets if and only if it is a member of each of the sets.

$$\forall x, s_1, s_2\ \ x \in (s_1 \cap s_2)) \Leftrightarrow (\, x \in s_1 \wedge x \in s_2)$$

8. An object is a member of the union of two sets if and only if it is a member of either set.

$$\forall x, s_1, s_2\ \ x \in (s_1 \cup s_2)) \Leftrightarrow (\, x \in s_1 \vee x \in s_2)$$

[4] Represent the following sentences in first-order logic

a) Not all students take both History and Biology.
b) Only one student failed in History.
c) The best score in History was better than the best score in Biology.
d) Every person who dislikes all vegeterians is smart.
e) No person likes a smart vegeterian.
f) There is a woman who likes all men who are vegeterians.
g) There is a barber who shaves all men in town who do not shave themselves.
h) No person likes a professor unless the professor is smart.

a. Not all students take both History and Biology

$\neg\forall$ x student(x) $\Rightarrow$ [takes(x, History) $\wedge$ takes(x, Biology)]

b. Only one student failed in history.

$\exists$ x $\forall$ y student(x) $\wedge$ fails(x, History) $\wedge$ student(y) $\wedge$ fails(y, History) $\Rightarrow$ x = y

c. The best score in History was better than the best score in Biology.

$\exists$ x $\forall$ y score(x, History) $\Rightarrow$ score(y, Biology)

d. Every person who dislikes all vegetarians is smart.

$\forall$ x $\forall$ y person(x) $\wedge$ (vegetarian(y) $\Rightarrow$ dislikes(x,y) $\Rightarrow$ smart(y)

e. No person likes a smart vegetarian.

$\forall$ x $\forall$ y person(x) $\wedge$ smart(y) $\wedge$ vegetarian(y) $\Rightarrow$ $\neg$likes(x,y)

f. There is a woman who likes all men who are not vegetarian.

$\exists$ x $\forall$y woman(x) $\wedge$ man(y) $\wedge$ $\neg$vegetarian(y)$\Rightarrow$ likes(x,y)

g. There is a barber who shaves all men in town who are not vegetarian.

$\exists$ x $\forall$ y barber(x) $\wedge$ man(y) $\wedge$ $\neg$ shaves(y) $\Rightarrow$ shaves(x,y)

h. No person likes a professor unless the professor is smart.

$\forall$ x $\forall$ y person(x) $\wedge$ professor(y) $\wedge\neg$smart(y) $\Rightarrow$ $\neg$likes(x,y)