

1. Inference

■ “If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.”

■ Can you prove it’s mythical? Magical? Horned?

Solution:

1. $(Mythical \Rightarrow Immortal) \wedge (\neg Mythical \Rightarrow Mortal \wedge Mammal)$
2. $(Immortal \vee Mammal \Rightarrow Horned)$
3. $(Horned \Rightarrow Magical)$
4. $Mythical \Rightarrow Immortal$ *AE(1)*
5. $\neg Mythical \Rightarrow Mortal \wedge Mammal$ *AE(1)*
6. $\neg Mythical \vee Immortal$ *IE(4)*
7. $\neg \neg Mythical \vee (Mortal \wedge Mammal)$ *IE(5)*
8. $Immortal \vee (Mortal \wedge Mammal)$ *Resolution(6,7)*
9. $(Immortal \vee Mortal) \wedge (Immortal \vee Mammal)$ *DL(8)*
10. $(Immortal \vee Mammal)$ *AE(9)*
11. $Horned$ *MP(2,10)*
12. $Magical$ *MP(3,11)*

2. Convert to $\sim((a \Leftrightarrow b) \vee c)$ to CNF

Solution

$$\neg(((a \Rightarrow b) \wedge (b \Rightarrow a)) \vee c)$$

$$\neg(((\neg a \vee b) \wedge (\neg b \vee a)) \vee c)$$

$$\neg((\neg a \vee b) \wedge (\neg b \vee a)) \wedge \neg c$$

$$(\neg(\neg a \vee b) \vee \neg(\neg b \vee a)) \wedge \neg c$$

$$((\neg\neg a \wedge \neg b) \vee (\neg\neg b \wedge \neg a)) \wedge \neg c$$

$$((a \wedge \neg b) \vee (b \wedge \neg a)) \wedge \neg c$$

$$((a \vee (b \wedge \neg a)) \wedge (\neg b \vee (b \wedge \neg a))) \wedge \neg c$$

$$((a \vee b) \wedge (a \vee \neg a)) \wedge ((\neg b \vee b) \wedge (\neg b \vee \neg a)) \wedge \neg c$$

$$(a \vee b) \wedge (\neg b \vee \neg a) \wedge \neg c$$

3. Which of the following are entailed by the sentence $(A \vee B) \wedge (\neg C \vee \neg D \vee E)$?

- i. $(A \vee B)$
- ii. $(A \vee B \vee C) \wedge (B \wedge C \wedge D \Rightarrow E)$
- iii. $(A \vee B) \wedge (\neg D \vee E)$

Solution

(9) Which of the following are entailed by the sentence $(A \vee B) \wedge (\neg C \vee \neg D \vee E)$?

- i. $(A \vee B)$
 ENTAILED: simple AND-elimination.
- ii. $(A \vee B \vee C) \wedge (B \wedge C \wedge D \Rightarrow E)$
 ENTAILED: $(B \wedge C \wedge D \Rightarrow E)$ is equivalent to $(\neg B \vee \neg C \vee \neg D \vee E)$, so this simply weakens the clause by introducing another disjunct.
- iii. $(A \vee B) \wedge (\neg D \vee E)$
 NOT ENTAILED: this removes the $\neg C$ literal, which strengthens the clause.