Assignment

(1) A logician was captured by a mafia group, detained in a cell room. There were two guards beside the room. One is honest and the other is dishonest. There are two exits from the room, one to freedom and the other to death. The chief of the group said to him, “You can ask a guard a question only once, and go through either exit depending on the answer”. After a while the logician worked out a question, and went to freedom. What is the question?

Note. The dishonest guard replies the opposite truth value to a question of a logical formula, whereas the honest respond the truth value itself. The logician cannot know which guard he is asking.

Proposition A : The asked guard is honest
Proposition B : Door 1 is to freedom

A simple question like A’? (Are you dishonest?), or B? (Is door 1 to freedom?) does not help. The question must be a composite one, such as A ^ B’? This does not work. Why?

The correct question Q is given by $Q=A\wedge B \vee A'\wedge B'$. The following is a proof. Let R be the response from the guard. R is given by $R=A\rightarrow Q \wedge A'\rightarrow Q'$. Then R is simplified by a logical calculus as follows:

$$R = (A' \vee Q)\wedge(A \vee Q') = A\wedge Q \vee A'\wedge Q' = A\wedge B \vee A'\wedge(A\wedge B' \vee A'\wedge B)$$
$$= A\wedge B \vee A'\wedge B = B$$

That is, R is equal to B. Thus if response is yes, door 1 is to freedom. If response is no, door 1 is not to freedom, meaning door 2 is to freedom, from the assumption that exactly one door is to freedom.

Now we generalize the problem slightly by introducing two propositions B1 and B2 as follows:

B1 : door 1 is to freedom
B2 : door 2 is to freedom

Substituting B1 for B in the above proof yields R=B1. We have the assumption $B1\vee B2$ is true. There is a possibility that both B1 and B2 are true. We have the following proof chart for the correct conclusion.

$R = B1,\ B1 \vee B2$

$R \to B1 \wedge R' \to B1',\ B1' \to B2$

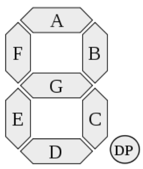
$R \to B1 \wedge R' \to B2$

That is, if answer is yes, take door 1, and if answer is no take door 2.

Now we generalize the problem further into three doors. Work out correct questions, and work out a correct escape strategy. Prove your escape strategy.
Hint. Use B1, B2 and B3 where Bi is “door i is to freedom”. Also assume $B1 \vee B2 \vee B3$ is true. Another hint. Is one question enough for escape?

(2) A microwave oven has a 4-digit 7-segment LED display that can show numbers, but also needs to show the words “ON”, “OFF”, “DONE” and “OPEN”. So, as well as the digits from 0 to 9, each digit needs to be able to show the 6 different letters on a display. You need to design a logic network for the LED display with 4 inputs (a binary number from 0 to 15) and 7 outputs (one for each segment of the display).

The layout of a seven-segment display is shown on the right. For example, the number 0 can be displayed by switching on all the segments except the middle one (G).



The input will be four bits, $b_3b_2b_1b_0$. For example, the input 0000 should display the number 0, and 1001 should display 9. The seven-segment patterns to use for the digits can be found at http://en.wikipedia.org/wiki/7_segment . However, you need to add the following six letters for the inputs 1010, 1011, 1100, 1101, 1110 and 1111 respectively:

For the assignment, you are required to work out 7 minimal functions to calculate each of the seven segments, A to G, for the 16 possible input combinations. Show the 7 corresponding Karnaugh maps, and express the functions as a formula and network. Trace your network by the binary input for one of letters of your choice.