K-Map

1. Minimise the following problems using the Karnaugh maps method.

   (a) Z = f(A,B,C) = $\bar{A}\bar{B}\bar{C} + \bar{A}B + AB\bar{C} + AC$

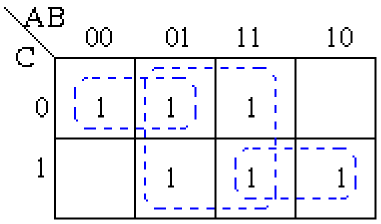
   (b) Z = f(A,B,C) = $\bar{A}B + B\bar{C} + BC + A\bar{B}\bar{C}$

Answer.

(a) Z = f(A,B,C) = $\bar{A}\bar{B}\bar{C} + \bar{A}B + AB\bar{C} + AC$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + A\bar{B}C + ABC$$

(000   010   011   110   101   111)



(The sequence must be kept as 00 01 11 10 so that only one variable changed each time)

By using the rules of simplification and ringing of adjacent cells in order to make as many variables redundant, the minimised result obtained is $\bar{A}\bar{C}$ +B + AC.

The first rectangle is $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} = \bar{A}\bar{C}(\bar{B}+B) = \bar{A}\bar{C}$

The second rectangle is $\bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC = \bar{A}B + AB = B$

The third rectangle is $ABC + A\bar{B}C = AC$

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Tip:

$Z = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + A\bar{B}C + ABC$

$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC + ABC + A\bar{B}C$

=first rectangle + second rectangle + third rectangle

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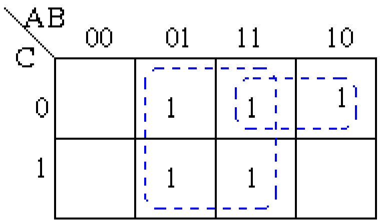
(b) Z = f(A,B,C) = $\bar{A}B + B\bar{C} + BC + A\bar{B}\bar{C}$



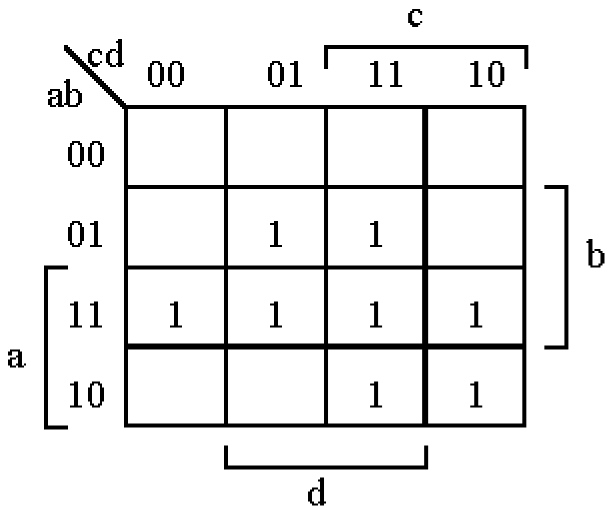
By using the rules of simplification and ringing of adjacent in order to make as many variables redundant, the minimised result obtained is B + A$\overline{C}$

2. Given the following 4-input Boolean expression

$$F(A,B,C,D) = \overline{A}B\overline{C}D + \overline{A}BCD + AB\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + ABC\overline{D} + A\overline{B}CD + A\overline{B}C\overline{D}...$$

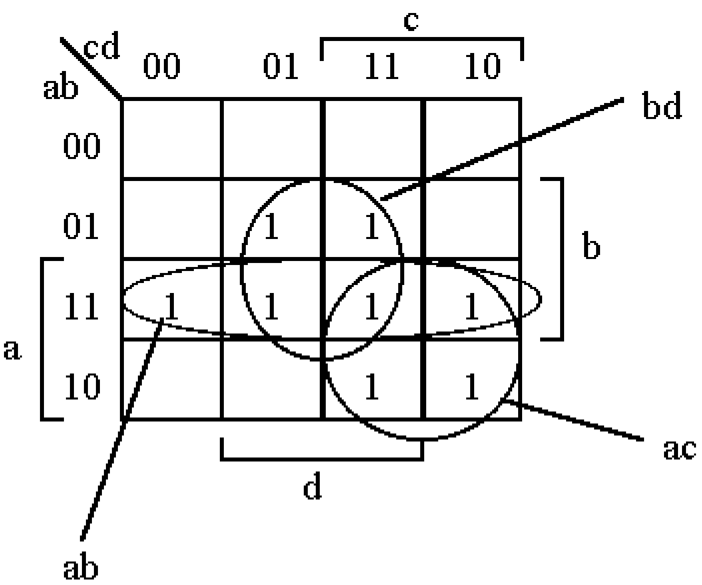
Simplify it using K-map.

Answer:



RULE: Minimization is achieved by drawing the smallest possible number of circles, each containing the largest possible number of 1s.

Grouping the 1s together results in the following.

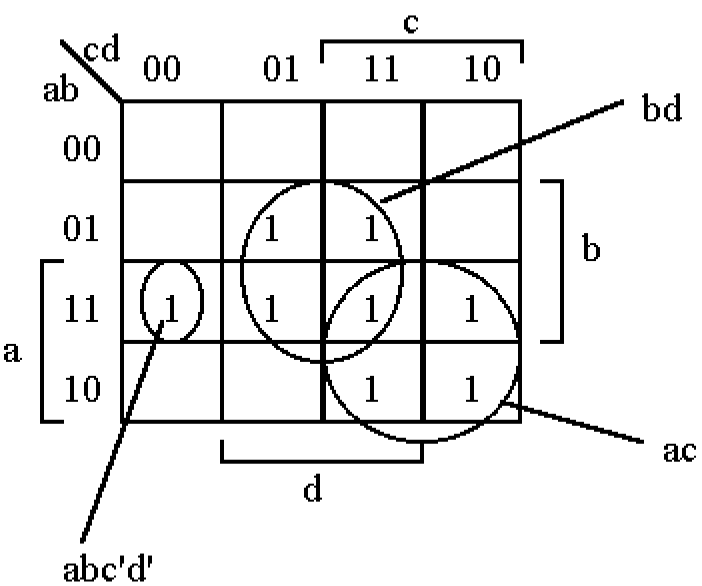


The expression for the groupings above is

q = bd + ac + ab

This expression requires 3 2-input **and** gates and 1 3-input **or** gate.

We could have accounted for all the 1s in the map as shown below, but that results in a more complex expression requiring a more complex gate.



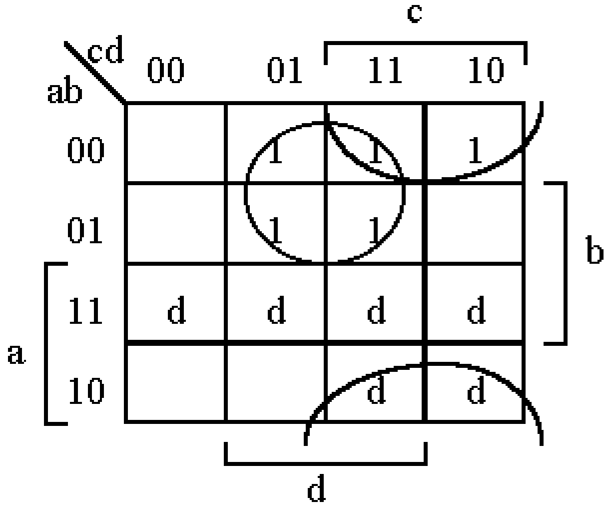
The expression for the above is **bd + ac + abc'd'**. This requires 2 2-input **and** gates, a 4-input **and** gate, and a 3 input **or** gate. Thus, one of the **and** gates is more complex (has two additional inputs) than required above. Two inverters are also needed.

3. Sometimes we do not care whether a 1 or 0 occurs for a certain set of inputs. It may be that those inputs will never occur so it makes no difference what the output is. For example, we might have a BCD (binary coded decimal) code which consists of 4 bits to encode the digits 0 (0000) through 9 (1001). The remaining codes (1010 through 1111) are not used. If we had a truth table for the prime numbers 0 through 9, it would be

abcd	p
0000	0
0001	1
0010	1
0011	1
0100	0
0101	1
0110	0
0111	1
1000	0
1001	0
1010	d
1011	d
1100	d
1101	d
1110	d
1111	d

The d's in the above stand for "don't care", we don't care whether a 1 or 0 is the value for that combination of inputs because (in this case) the inputs will never occur. Simply this prime number function using K-map.

Answer.

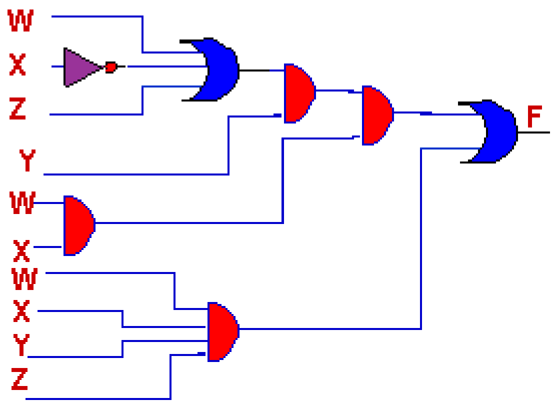


The circle made entirely of 1s corresponds to the expression **a'd** and the combined 1 and d circle (actually a combination of arcs) is **b'c**. Thus, if the disallowed input 1011 did occur, the output would be 1 but if the disallowed input 1100 occurs, its output would be 0. The minimized expression is

p = a'd + b'c

Notice that if we had ignored the ds and only made a circle around the 2 1s, the resulting expression would have been more complex, **a'b'c** instead of **b'c**.

4. Eason has been working with a logic circuit. He has a circuit that works but he suspects that it can be made simpler. Here is the circuit.



(a) Determine the truth table for the function implemented in this circuit. Fill in the truth table below.

W	X	Y	Z	F
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

WX \ YZ	00	01	11	10
00				
01				
11				
10				

(b) Using the Karnaugh Map, get the simplest sum-of-products form for this function

(c) Draw the circuit diagram using ANDs, ORs and NOTs.

(d) Draw the circuit diagram using all NANDs.

Answer.

(a)

$$\begin{aligned} F &= (W + \overline{X} + Z) \cdot Y \cdot (WX) + WXYZ \\ &= WXY(W + \overline{X} + Z) + WXYZ \\ &= WWXY + WX\overline{X}Y + WXYZ + WXYZ \\ &= WXY + WXYZ \\ &= WXY \end{aligned}$$

So the truth table is

W	X	Y	Z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

(b) Using the Karnaugh Map, get the simplest sum-of-products form for this function

		YZ			
		00	01	11	10
WX	00				
	01				
	11			1	1
	10				

$$F = WXYZ + WXY\overline{Z}$$
$$= WXY$$

(c)Draw the circuit diagram using ANDs, ORs and NOTs.

$$F = WXY$$

(d)Draw the circuit diagram using all NANDs.

$$F = \overline{\left(\overline{WXY}\right)\left(\overline{WXY}\right)}$$

5. Problem B.8 of textbook.

B.8 A combinational circuit is used to control a seven-segment display of decimal digits, as shown in Figure B.34. The circuit has four inputs, which provide the four-bit code used in packed decimal representation ($0_{10} = 0000, \ldots, 9_{10} = 1001$). The seven outputs define which segments will be activated to display a given decimal digit. Note that some combinations of inputs and outputs are not needed.

a. Develop a truth table for this circuit.
b. Express the truth table in SOP form.
c. Express the truth table in POS form.
d. Provide a simplified expression.

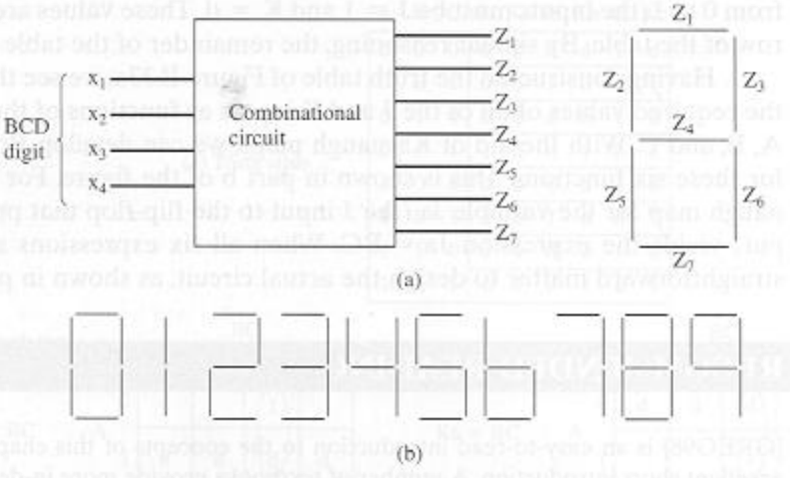


Figure B.34 Seven-Segment LED Display Example

Answer

B.8 **a.**

X_1	X_2	X_3	X_4	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7
0	0	0	0	1	1	1	0	1	1	1
0	0	0	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0	1
0	0	1	1	1	0	1	1	0	1	1
0	1	0	0	0	1	1	1	0	1	0
0	1	0	1	1	1	0	1	0	1	1
0	1	1	0	0	1	0	1	1	1	1
0	1	1	1	1	0	1	0	0	1	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	0
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

b. All of the terms have the form illustrated as follows:

$$Z_5 = \overline{X_1 X_2 X_3 X_4} + \overline{X_1 X_2}\, X_3\, \overline{X_4} + \overline{X_1}\, X_2 X_3\, \overline{X_4} + \overline{X_1}\, X_2 X_3 X_4$$

c. Whereas the SOP form lists all combinations that produce an output of 1, the POS lists all combinations that produce an output of 0.

b.

$$Z_1 = \overline{X}_1\overline{X}_2\overline{X}_3\overline{X}_4 + \overline{X}_1\overline{X}_2 X_3\overline{X}_4 + \overline{X}_1\overline{X}_2 X_3 X_4 + \overline{X}_1 X_2\overline{X}_3 X_4 + \overline{X}_1 X_2 X_3 X_4 + X_1\overline{X}_2\overline{X}_3\overline{X}_4 + X_1\overline{X}_2\overline{X}_3 X_4$$

$$Z_2 = \overline{X}_1\overline{X}_2\overline{X}_3\overline{X}_4 + \overline{X}_1 X_2\overline{X}_3\overline{X}_4 + \overline{X}_1 X_2\overline{X}_3 X_4 + \overline{X}_1 X_2 X_3\overline{X}_4 + X_1\overline{X}_2\overline{X}_3\overline{X}_4 + X_1\overline{X}_2\overline{X}_3 X_4$$

c.

$$Z_1 = \overline{\left(\overline{X}_1\overline{X}_2\overline{X}_3 X_4\right)}\,\overline{\left(\overline{X}_1 X_2\overline{X}_3\overline{X}_4\right)}\,\overline{\left(\overline{X}_1 X_2 X_3\overline{X}_4\right)}\,\overline{\left(X_1\overline{X}_2 X_3\overline{X}_4\right)}\,\overline{\left(X_1\overline{X}_2 X_3 X_4\right)}\,\overline{\left(X_1 X_2\overline{X}_3\overline{X}_4\right)}\,\overline{\left(X_1 X_2\overline{X}_3 X_4\right)}\,\overline{\left(X_1 X_2 X_3\overline{X}_4\right)}\,\overline{\left(X_1 X_2 X_3 X_4\right)}$$

$$= \left(\overline{X}_1 + \overline{X}_2 + \overline{X}_3 + X_4\right)\left(\overline{X}_1 + X_2 + \overline{X}_3 + \overline{X}_4\right)\left(\overline{X}_1 + X_2 + X_3 + \overline{X}_4\right)\left(X_1 + \overline{X}_2 + X_3 + \overline{X}_4\right)\left(X_1 + \overline{X}_2 + X_3 + X_4\right)\left(X_1 + X_2 + \overline{X}_3 + \overline{X}_4\right)\left(X_1 + X_2 + \overline{X}_3 + X_4\right)\left(X_1 + X_2 + X_3 + \overline{X}_4\right)\left(X_1 + X_2 + X_3 + X_4\right)$$

d.

The K-map for Z_1

		X_1X_2			
		00	01	11	10
X_3X_4	00	1			1
	01		1		1
	11	1	1		
	10	1			

$$\begin{aligned} Z_1 &= \left(\overline{X_1}\,\overline{X_2}\,\overline{X_3}\,\overline{X_4} + \overline{X_1}\,\overline{X_2}X_3\overline{X_4}\right) + \left(\overline{X_1}\,\overline{X_2}X_3X_4 + \overline{X_1}\,\overline{X_2}X_3\overline{X_4}\right) \\ &\quad + \left(\overline{X_1}X_2\overline{X_3}X_4 + \overline{X_1}X_2X_3X_4\right) + \left(X_1\overline{X_2}\,\overline{X_3}\,\overline{X_4} + X_1\overline{X_2}\,\overline{X_3}X_4\right) \\ &= \overline{X_1}\,\overline{X_2}\,\overline{X_4} + \overline{X_1}\,\overline{X_2}X_3 + \overline{X_1}X_2X_4 + X_1\overline{X_2}\,\overline{X_3} \end{aligned}$$