

### Lab 3

#### 1) Complete

- a)  $p \wedge \neg p =$  \_\_\_\_\_
- b)  $p \vee \neg p =$  \_\_\_\_\_
- c)  $p \vee T =$  \_\_\_\_\_
- d)  $p \wedge F =$  \_\_\_\_\_
- e)  $p \vee p =$  \_\_\_\_\_
- f)  $\neg (p \vee q) =$  \_\_\_\_\_

#### 2) the propositions p NAND q and p NOR q are denoted by $p|q$ and $p \downarrow q$ respectively,

- a) construct a TT for the logical operator NAND and for NOR
- b) show that  $p|q$  is logically equivalent to  $\neg(p \wedge q)$
- c) show that  $p \downarrow q$  is logically equivalent to  $\neg(p \vee q)$
- d) show that  $p \downarrow p$  is logically equivalent to  $\neg p$
- e) show that  $(p \downarrow q) \downarrow (p \downarrow q)$  is logically equivalent to  $p \vee q$
- f) show that  $p|q$  and  $q|p$  are logical equivalent
- g) show that  $p|(q|r)$  and  $(p|q)|r$  are not equivalent

It is possible to define PL using signs ' $\neg$ ' and ' $\vee$ '.

- $(\alpha \& \beta)$  would be defined as  $\neg(\neg\alpha \vee \neg\beta)$ ,
- $(\alpha \rightarrow \beta)$  would be defined as  $(\neg\alpha \vee \beta)$ ,
- $(\alpha \leftrightarrow \beta)$  would be defined as  $\neg(\neg(\neg\alpha \vee \beta) \vee \neg(\neg\beta \vee \alpha))$ .

Similarly, we could instead use ' $\neg$ ' and '&'.

- $(\alpha \vee \beta)$  would be defined as  $\neg(\neg\alpha \& \neg\beta)$ ,
- $(\alpha \rightarrow \beta)$  would be defined as  $\neg(\alpha \& \neg\beta)$ ,
- $(\alpha \leftrightarrow \beta)$  would be defined as  $(\neg(\alpha \& \neg\beta) \& \neg(\beta \& \neg\alpha))$ .

Another way, PL" is defined using ' $\downarrow$ ', *Sheffer dagger(NOR)*.

- $\neg\alpha$  is defined as  $(\alpha \downarrow \alpha)$ ;
- $(\alpha \vee \beta)$  is defined as  $((\alpha \downarrow \beta) \downarrow (\alpha \downarrow \beta))$ ;
- $(\alpha \& \beta)$  is defined as  $((\alpha \downarrow \alpha) \downarrow (\beta \downarrow \beta))$ ;
- and so on for the other operators

### Answer of lab 3

#### 1) Complete

- a)  $p \wedge \neg p =$  \_\_\_\_\_F\_\_\_\_\_
- b)  $p \vee \neg p =$  \_\_\_\_\_T\_\_\_\_\_
- c)  $p \vee T =$  \_\_\_\_\_T\_\_\_\_\_
- d)  $p \wedge F =$  \_\_\_\_\_F\_\_\_\_\_
- e)  $p \vee p =$  \_\_\_\_\_P\_\_\_\_\_

f)  $\neg(p \vee q) = \underline{\quad} \neg p \wedge \neg q \underline{\quad}$

2) the propositions p NAND q and p NOR q are denoted by  $p|q$  and  $p \downarrow q$  respectively,

a) construct a TT for the logical operator NAND and for NOR

p	q	$p q$	$p \downarrow q$
T	T	F	F
T	F	T	F
F	T	T	F
F	F	T	T

b) show that  $p|q$  is logically equivalent to  $\neg(p \wedge q)$

p	q	$p q$	$p \wedge q$	$\neg(p \wedge q)$	$(p q) \Leftrightarrow \neg(p \wedge q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	T	F	T	T
F	F	T	F	T	T

c) show that  $p \downarrow q$  is logically equivalent to  $\neg(p \vee q)$

p	q	$p \downarrow q$	$p \vee q$	$\neg(p \vee q)$	$(p \downarrow q) \Leftrightarrow \neg(p \vee q)$
T	T	F	F	F	T
T	F	F	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T

d) show that  $p \downarrow p$  is logically equivalent to  $\neg p$

p	$p \downarrow p$	$\neg p$	$p \downarrow p \Leftrightarrow \neg p$
T	F	F	T
F	T	T	T

e) show that  $(p \downarrow q) \downarrow (p \downarrow q)$  is logically equivalent to  $p \vee q$

p	q	$p \downarrow q$	$(p \downarrow q) \downarrow (p \downarrow q)$	$p \vee q$	$(p \downarrow q) \downarrow (p \downarrow q) \Leftrightarrow p \vee q$
T	T	F	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T

f) show that  $p|q$  and  $q|p$  are logical equivalent

p	q	$p q$	$q p$	$(p q) \Leftrightarrow (q p)$
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T

g) show that  $p|(q|r)$  and  $(p|q)|r$  are not equivalent

p	q	r	$(p q)$	$(p q) r$	$(q r)$	$p (q r)$	$p (q r) \Leftrightarrow (p q) r$
T	T	T	F	F	F	F	T
T	T	F	F	F	F	F	T
T	F	T	T	F	F	F	T
T	F	F	T	F	F	F	T
F	T	T	T	F	F	F	T

F	T	F					T
F	F	T					T
F	F	F					T