lab 2

1) For each of the following compound statements, first identify the primitive statements p, q, r, etc. of which it is composed. Then express the statement symbolically.

a. If n is an integer, then either n is even or n is odd.
b. If an integer is prime and is greater than 2, then the integer is odd.
c. If n is not negative and its square is less than 4, then either n is zero or n is positive and less than 2.
d. If an integer is even and greater than 2, then it is not prime.

2) prove the following

a) $(p \Rightarrow q) \equiv ((\sim p) \vee q)$
b) $(p \Rightarrow q) \equiv ((\sim q) \Rightarrow (\sim p))$
c) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
d) $\sim(p \Leftrightarrow q) \equiv ((p \wedge \sim q) \vee (q \wedge \sim p))$

3) construct truth table for

$(z' \vee x) \wedge ((x \wedge y) \vee z) \wedge (z' \vee y) \Leftrightarrow x \wedge y$

explain the result

4) calculate TT for $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$ and determine its type

5) Let p and q be the propositions

p : I bought a lottery ticket this week.
q : I won the million dollar jackpot on Friday.

Expression each of these propositions as an English sentence.

e) $p \leftrightarrow q$
f) $\neg p \rightarrow \neg q$

6). Let p, q, and r be the propositions

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

7). Determine whether these biconditionals are true or false.
d) $0 > 1$ if and only if $2 > 1$.

8). State the converse, contrapositive, and inverse of each of these conditional statements.
a) If it snows tonight, then I will stay at home.
b) I go to the beach whenever it is a sunny summer day.
c) When I stay up late, it is necessary that I sleep until noon.

9. Construct a truth table for each of these compound propositions.
e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

10). An ancient Sicilian legend says that the barber in a remote town who can be reached only by traveling a dangerous mountain road shaves those people, and only those people, who do not shave themselves. Can there be such a barber?

11). Even though we normally use "implies" and "if ..., then" to describe implication, other word orders and phrases often arise in practice, as in the example below. Let p, q, and r be the propositions:

p = "the flag is set,"
q = "I=0,"
r = "subroutine S is completed."

Translate each of the following propositions into symbols, using the letters p. q. r and logical connectives.
a) If the flag is set, then I=0.
b) The flag is set and I=0 if subroutine S is not completed.
c) Subroutine S is completed if and only if I=0 and flag is set.

Answer of lab 2

1) For each of the following compound statements, first identify the primitive statements p, q, r, etc. of which it is composed. Then express the statement symbolically.

a. If n is an integer, then either n is even or n is odd.

P: n is an integer; Q: n is even; R: n is odd

$P \rightarrow (Q \oplus R)$

b. If an integer is prime and is greater than 2, then the integer is odd.

P: the integer is prime; Q: the integer > 2; R: the integer is odd

$(P \wedge Q) \rightarrow R$

c. If n is not negative and its square is less than 4, then either n is zero or n is positive and less than 2.

P: n < 0; Q: n2 < 4; R: n = 0; S: n > 0; T: n < 2

$(\neg P \wedge Q) \rightarrow R \oplus (S \wedge T)$

d. If an integer is even and greater than 2, then it is not prime.

P: integer is even; Q: integer > 2; R: integer is prime

$(P \wedge Q) \rightarrow \neg R$

2) $(p \Rightarrow q) \equiv ((\sim p) \vee q)$

p	q	(p⇒q)	(~p)	((~p) ∨ q)	(p⇒q) ⇔ ((~p) ∨ q)
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$(p \Rightarrow q) \equiv ((\sim q) \Rightarrow (\sim p))$

p	q	(p⇒q)	(~q)	(~p)	((~q) ⇒ (~p))	(p⇒q) ⇔ ((~p) ∨ q)
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

p	q	(p∨q)	~ (p∨q)	(~p)	(~q)	(~p) ∧ (~q)	~(p∨q) ⇔ (~p) ∧ (~q)
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

$\sim(p \Leftrightarrow q) \equiv ((p \wedge \sim q) \vee (q \wedge \sim p))$

p	q	(p⇔q)	~(p⇔q) s1	~q	p∧~q	~p	q∧~p	(p∧~q) ∨(q∧~p) s2	s1⇔s2
T	T	T	F	F	F	F	F	F	T
T	F	F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T	T	T
F	F	T	F	T	F	T	F	F	T

3) construct truth table for (z' ∨ x) ∧ ((x∧y)∨z) ∧ (z'∨y) ⇔ x ∧ y

s2 s1

x	y	z	z'	z'∨x	x∧y	(x∧y)∨z	z'∨y	(z'∨x) ∧ ((x∧y)∨z) ∧ (z'∨y)	s1⇔s2
T	T	T	F	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T	T
T	F	T	F	T	F	T	F	F	T
T	F	F	T	T	F	F	T	F	T
F	T	T	F	F	F	T	T	F	T
F	T	F	T	T	F	F	T	F	T
F	F	T	F	F	F	T	F	F	T
F	F	F	T	T	F	F	T	F	T

4) calculate TT for [p∨(q∧r)]⇔[(p∨q) ∧ (p∨r)] and determine its type

p	q	r	(q∧r)	p∨(q∧r)	(p∨q)	(p∨r)	(p∨q) ∧ (p∨r)	p∨(q∧r)⇔(p∨q) ∧ (p∨r)
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

Tautology

5. Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

Expression each of these propositions as an English sentence.

e) $p \leftrightarrow q$

I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

f) $\neg p \rightarrow \neg q$

If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

6. Let p, q, and r be the propositions

p : Grizzly bears have been seen in the area.

q : Hiking is safe on the trail.

r : Berries are ripe along the trail.

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

$r \land \neg p$

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

$\neg p \land q \land r$

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

$r \rightarrow (q \leftrightarrow \neg p)$

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

$\neg q \land \neg p \land r$

e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

$(q \rightarrow (\neg r \land \neg p)) \land \neg((\neg r \land \neg p) \rightarrow q)$

f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$(p \land r) \rightarrow \neg q$

7. Determine whether these biconditionals are true or false.

d) $0 > 1$ if and only if $2 > 1$.

This is $F \leftrightarrow T$, which is false.

8. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

Converse: If I stay home, then it will snow tonight.

Contrapositive: If I do not stay home, then it will not snow tonight.

Inverse: If it does not snow tonight, then I will not stay home.

b) I go to the beach whenever it is a sunny summer day.

Converse: Whenever I go to the beach, it is a sunny summer day.

Contrapositive: Whenever I do not go to the beach, it is not a sunny summer day.

Inverse: Whenever it is not a sunny day, I do not go to the beach.

c) When I stay up late, it is necessary that I sleep until noon.

Converse: If I sleep until noon, then I stayed up late.

Contrapositive: If I do not sleep until noon, then I did not stay up late.

Inverse: If I don't stay up late, then I don't sleep until noon.

9. Construct a truth table for each of these compound propositions.
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

10. An ancient Sicilian legend says that the barber in a remote town who can be reached only by traveling a dangerous mountain road shaves those people, and only those people, who do not shave themselves. Can there be such a barber?

No. This is a classical paradox. If such a barber existed, who would shave the barber? If the barber shaved himself, then he would be violating the rule that he shaves only those people who do not shave themselves. On the other hand, if he does not shave himself, then the rule says that he must shave himself. Neither is possible, so there can be no such barber.

11. Even though we normally use "implies" and "if ..., then" to describe implication, other word orders and phrases often arise in practice, as in the example below. Let p, q, and r be the propositions:

p = "the flag is set,"
q = "I=0,"
r = "subroutine S is completed."

Translate each of the following propositions into symbols, using the letters p. q. r and logical connectives.

a) If the flag is set, then I=0.

$p \rightarrow q$

b) The flag is set and I=0 if subroutine S is not completed.

$\neg r \rightarrow p \wedge q$

c) Subroutine S is completed if and only if I=0 and flag is set.

$r \leftrightarrow q \wedge p$