

[1] Write a BNF for propositional logic

<i>Sentence</i>	\rightarrow	<i>AtomicSentence</i> <i>ComplexSentence</i>
<i>AtomicSentence</i>	\rightarrow	True False <i>Symbol</i>
<i>Symbol</i>	\rightarrow	P Q R ...
<i>ComplexSentence</i>	\rightarrow	\neg <i>Sentence</i> (<i>Sentence</i> \wedge <i>Sentence</i>) (<i>Sentence</i> \vee <i>Sentence</i>) (<i>Sentence</i> \Rightarrow <i>Sentence</i>) (<i>Sentence</i> \Leftrightarrow <i>Sentence</i>)

[2] Consider the following sentences and decide for each if it is valid, unsatisfiable, or neither. Verify your decisions using truth tables

- a. *Smoke* \Rightarrow *Smoke*
- b. *Smoke* \Rightarrow *Fire*
- c. (*Smoke* \Rightarrow *Fire*) \Rightarrow (\neg *Smoke* \Rightarrow \neg *Fire*)
- d. *Smoke* \vee *Fire* \vee \neg *Fire*
- e. ((*Smoke* \wedge *Heat*) \Rightarrow *Fire*) \Leftrightarrow ((*Smoke* \Rightarrow *Fire*) \vee (*Heat* \Rightarrow *Fire*))
- f. (*Smoke* \Rightarrow *Fire*) \Rightarrow ((*Smoke* \wedge *Heat*) \Rightarrow *Fire*)

a. *Smoke* \Rightarrow *Smoke*

<i>Smoke</i>	<i>Smoke</i> \Rightarrow <i>Smoke</i>
T	T
F	T

Valid

b. $Smoke \Rightarrow Fire$

<i>Smoke</i>	<i>Fire</i>	<i>Smoke \Rightarrow Fire</i>
T	T	T
T	F	F
F	T	T
F	F	T

Satisfiable

c. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

<i>Smoke</i>	<i>Fire</i>	<i>Smoke \Rightarrow Fire</i>	\neg <i>Smoke \Rightarrow Fire</i>	<i>(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)</i>
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Satisfiable

d. $Smoke \vee Fire \vee \neg Fire$

<i>Smoke</i>	<i>Fire</i>	<i>Fire \vee \neg Fire</i>	<i>Smoke \vee Fire \vee \neg Fire</i>
T	T	T	T
T	F	T	T
F	T	T	T
F	F	T	T

Valid

e. $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

<i>S</i>	<i>F</i>	<i>H</i>	<i>S \wedge H</i>	<i>(S \wedge H) \Rightarrow F</i>	<i>S \Rightarrow F</i>	<i>H \Rightarrow F</i>	<i>(S \Rightarrow F) \vee (H \Rightarrow F)</i>	<i>The sentence</i>
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	T	T	F
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	F	T	T

Satisfiable

f. $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$

<i>S</i>	<i>F</i>	<i>H</i>	$S \wedge H$	$(S \wedge H) \Rightarrow F$	$S \Rightarrow F$	<i>The sentence</i>
T	T	T	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	F	F	T
T	F	F	F	T	F	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Valid